Supplemental Appendix to:
Imperfect Competition and Rents in Labor and Product Markets: The Case of the Construction Industry

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A. Model Derivations

1. The Composite Production Function

We now derive equation (9). To do so, we express revenues and costs as functions of Q_{jt} so as to separate the joint maximization into two steps: In the first step, we find the optimal combination (K_{jt}, L_{jt}) for each Q_{jt}. In the second step, we solve for the optimal Q_{jt}.

Recall that firms can rent capital at price p_K and hire labor at price $W_{jt} = U_{jt} L_{jt}^{\theta}$. The production function (in physical units) satisfies

$$Q_{jt} = \Omega_{jt} K_{jt}^{\beta_K} L_{jt}^{\beta_L}. \tag{A.1}$$

Intermediate inputs have constant price p_M and, due to the Leontief functional form, must satisfy $M_{jt} = Q_{jt}/\beta_M$. Given any production level Q_{jt}, the firm can find the most cost efficient combination (K_{jt}, L_{jt}) by solving the Hicksian cost-minimization problem,

$$\min_{(K_{jt}, L_{jt}): Q_{jt} = \Omega_{jt} K_{jt}^{\beta_K} L_{jt}^{\beta_L}} p_K K_{jt} + U_{jt} L_{jt}^{1+\theta}, \tag{A.2}$$

where $p_K K_{jt} + U_{jt} L_{jt}^{1+\theta}$ is the total cost of capital and labor. We now solve for the Hicksian demand for capital and labor using the Lagrangian,

$$\mathcal{L}_{jt} \equiv p_K K_{jt} + U_{jt} L_{jt}^{1+\theta} + \lambda_{jt}(Q_{jt} - \Omega_{jt} K_{jt}^{\beta_K} L_{jt}^{\beta_L}), \tag{A.3}$$

where λ_{jt} is the Lagrange multiplier. The first-order conditions for capital and labor, respectively, are as follows:

$$p_K = \lambda_{jt} \Omega_{jt} \beta_K K_{jt}^{\beta_K - 1} L_{jt}^{\beta_L}, \tag{A.4}$$

$$(1+\theta) U_{jt} L_{jt}^{\theta} = \lambda_{jt} \Omega_{jt} \beta_L K_{jt}^{\beta_K} L_{jt}^{\beta_L - 1}. \tag{A.5}$$

These equations lead to the optimal choice of capital as a function of labor:

$$K_{jt} = \frac{\beta_K}{\beta_L} \frac{(1+\theta)}{p_K} U_{jt} L_{jt}^{1+\theta}. \tag{A.6}$$

Substituting equation (A.6) into equation (A.1), the inverse Hicksian demand is

$$Q_{jt} = \Omega_{jt} \left(\frac{\beta_K}{\beta_L} \frac{(1+\theta)}{p_K} U_{jt} L_{jt}^{1+\theta} \right)^{\beta_K} L_{jt}^{\beta_L} = \Phi_{jt} L_{jt}^{\rho}, \tag{A.7}$$

where we define $\rho \equiv (1+\theta)\beta_K + \beta_L$ and $\Phi_{jt} \equiv \Omega_{jt}\left(\frac{\beta_K}{\beta_L}\frac{(1+\theta)}{p_K}U_{jt}\right)^{\beta_K}$. Thus, $L_{jt} = (Q_{jt}/\Phi_{jt})^{1/\rho}$. Substituting into the first-order condition for intermediate inputs (equation 10), the Hicksian expenditure on intermediate inputs is

$$X_{jt} \equiv p_M M_{jt} = p_M Q_{jt}/\beta_M = \frac{p_M}{\beta_M}\Phi_{jt}L_{jt}^{\rho}. \tag{A.8}$$

Lastly, letting $\kappa_U \equiv \frac{\beta_K}{\beta_L}(1+\theta)+1$, total costs can be expressed purely in terms of labor as

$$W_{jt}L_{jt} + p_K K_{jt} + p_M M_{jt} = \kappa_U U_{jt} L_{jt}^{1+\theta} + \frac{p_M}{\beta_M}\Phi_{jt}L_{jt}^{\rho}. \tag{A.9}$$

2. Firm's Behavior in the Private Product Market

We now derive equation (13) and several related results on firm behavior in the private market. We assume a downward-sloping private product demand curve ($\epsilon > 0$) and increasing composite returns to labor ($\rho > 1$), consistent with the empirical evidence.

If $d = 0$, the firm's profit maximization problem is,

$$\max_{L_{0jt}} p_H\left(\Phi_{jt}L_{0jt}^{\rho}\right)^{1-\epsilon} - \kappa_U U_{jt}L_{0jt}^{1+\theta} - \frac{p_M}{\beta_M}\Phi_{jt}L_{0jt}^{\rho}, \tag{A.10}$$

where we substituted equations (9) and (A.9) into equation (8) for the case with $d = 0$. The profit-maximizing first-order condition is,

(A.11)

$$\frac{\partial \pi_{0jt}}{\partial L_{0jt}} \equiv \underbrace{p_H\Phi_{jt}^{1-\epsilon}(1-\epsilon)\rho L_{0jt}^{(1-\rho)\epsilon-(1-\rho)-\epsilon}}_{\text{MRP}} - \underbrace{\left(\kappa_U U_{jt}(1+\theta)L_{0jt}^{\theta} + \frac{p_M}{\beta_M}\Phi_{jt}\rho L_{0jt}^{-(1-\rho)}\right)}_{\text{MCL}} = 0.$$

This expression shows that L_{0jt} only varies across firms due to Φ_{jt} and U_{jt}. We now derive equation (13) and several implications. Equation (A.11) can be arranged as

(A.12)

$$\underbrace{\overbrace{(1-\epsilon)}^{\text{markup}^{-1}}\overbrace{p_H\Phi_{jt}^{-\epsilon}L_{0jt}^{-\rho\epsilon}}^{P_{jt}}\overbrace{\Phi_{jt}\rho L_{0jt}^{\rho-1}/\kappa_U}^{\text{MP}_{jt}}}_{\text{MRP}_{jt}} = \underbrace{\overbrace{(1+\theta)}^{\text{markdown}^{-1}}\overbrace{U_{jt}L_{0jt}^{\theta}}^{W_{jt}}}_{\text{MCL}_{jt}} + \underbrace{\overbrace{\frac{p_M}{\beta_M}}^{s_M P_{jt}}\overbrace{\Phi_{jt}\rho L_{0jt}^{\rho-1}/\kappa_U}^{\text{MP}_{jt}}}_{\text{marginal intermed. costs}},$$

which is the same as equation (19), where we use that $\beta_M = Q_{jt}/M_{jt}$ implies $\frac{p_M}{\beta_M} = \frac{p_M M_{jt}}{Q_{jt}} = \frac{p_M M_{jt}}{Q_{jt}P_{jt}}P_{jt} = \frac{X_{jt}}{R_{jt}}P_{jt}$.

We will now show that MRP is greater than MCL as L_{0jt} approaches zero. Multiplying marginal profits in (A.11) by $L_{0jt}^{\rho\epsilon+(1-\rho)}$, which is strictly positive, we have

(A.13)
$$\frac{\partial \pi_{0jt}}{\partial L_{0jt}} L_{0jt}^{\rho\epsilon+(1-\rho)} = \underbrace{p_H \Phi_{jt}^{1-\epsilon}(1-\epsilon)\rho}_{\text{MRP}\times L_{0jt}^{\rho\epsilon+(1-\rho)}} - \underbrace{\left(\kappa_U U_{jt}(1+\theta)L_{0jt}^{\theta+\rho\epsilon+(1-\rho)} + \frac{p_M}{\beta_M}\Phi_{jt}\rho L_{0jt}^{\rho\epsilon}\right)}_{\text{MCL}\times L_{0jt}^{\rho\epsilon+(1-\rho)}}.$$

Note that MRP $\times L_{0jt}^{\rho\epsilon+(1-\rho)}$ is constant with respect to L_{0jt} and positive. By contrast, given $\theta + \rho\epsilon + (1-\rho) > 0$, then MCL $\times L_{0jt}^{\rho\epsilon+(1-\rho)}$ converges to zero as L_{0jt} approaches zero. Thus, we have shown that $\lim_{L_{0jt}\to 0^+} \frac{\partial \pi_{0jt}}{\partial L_{0jt}} > 0$. As a result, it is always optimal to choose $L_{0jt} > 0$ if $\theta + \rho\epsilon + (1-\rho) > 0$.

Furthermore, multiplying both sides of equation (A.11) by L_{0jt}, we have

$$\text{(A.14)} \quad \frac{\partial \pi_{0jt}}{\partial L_{0jt}} \equiv p_H \Phi_{jt}^{1-\epsilon}(1-\epsilon)\rho L_{0jt}^{\rho(1-\epsilon)} - \kappa_U U_{jt}(1+\theta)L_{0jt}^{1+\theta} - \frac{p_M}{\beta_M}\Phi_{jt}\rho L_{0jt}^{\rho} = 0.$$

Recall that $\kappa_U U_{jt} L_{0jt}^{1+\theta} = \frac{\rho}{\beta_L} B_{0jt}$, $R_{0jt}^H = p_H \Phi_{jt}^{1-\epsilon} L_{0jt}^{\rho(1-\epsilon)}$, and $X_{0jt} = \frac{p_M}{\beta_M}\Phi_{jt} L_{0jt}^{\rho}$. Substituting, we have

$$\text{(A.15)} \quad (1-\epsilon) R_{0jt}^H = \frac{1+\theta}{\beta_L} B_{0jt} + X_{0jt}.$$

Similarly, if $d = 1$, the firm's profit maximization problem is,

$$\text{(A.16)} \quad \max_{L_{1jt}:\, \Phi_{jt} L_{1jt}^{\rho} \geq \overline{Q}^G} p_H \left(\Phi_{jt} L_{1jt}^{\rho} - \overline{Q}^G\right)^{1-\epsilon} - \kappa_U U_{jt} L_{1jt}^{1+\theta} - \frac{p_M}{\beta_M}\Phi_{jt} L_{1jt}^{\rho}.$$

The first-order condition is,

(A.17)
$$\frac{\partial \pi_{1jt}}{\partial L_{1jt}} \equiv p_H \Phi_{jt}(1-\epsilon)\rho \left(\Phi_{jt} L_{1jt}^{\rho} - \overline{Q}^G\right)^{-\epsilon} L_{1jt}^{-(1-\rho)} - \kappa_U U_{jt}(1+\theta) L_{1jt}^{\theta} - \frac{p_M}{\beta_M}\Phi_{jt}\rho L_{1jt}^{-(1-\rho)} = 0.$$

As $\Phi_{jt} L_{1jt}^{\rho}$ approaches $\overline{Q}^G$, $\left(\Phi_{jt} L_{1jt}^{\rho} - \overline{Q}^G\right)^{-\epsilon}$ approaches infinity while all other terms involving L_{1jt} approach constants. Thus, $\Phi_{jt} L_{1jt}^{\rho} > \overline{Q}^G$ is necessary to satisfy the equation. Since $Q_{1jt} = \Phi_{jt} L_{1jt}^{\rho}$, it follows that $Q_{1jt}^H = Q_{1jt} - \overline{Q}^G > 0$, so the winning firm always produces for the private market. Furthermore, it is always true that $\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=L_{0jt}} > 0$. Thus, Q_{djt} is larger if $d = 1$ than $d = 0$.

Multiplying both sides of equation (A.17) by L_{1jt} and replacing $R_{1jt}^H = p_H \left(\Phi_{jt} L_{1jt}^{\rho} - \overline{Q}^G\right)^{1-\epsilon}$,

$$\text{(A.18)} \quad R_{1jt}^H \Phi_{jt}(1-\epsilon) \left(\Phi_{jt} L_{1jt}^{\rho} - \overline{Q}^G\right)^{-1} L_{1jt}^{\rho} - \frac{1+\theta}{\beta_L} B_{1jt} - X_{1jt} = 0.$$

Since $Q^H_{1jt} = \Phi_{jt}L^{\rho}_{1jt} - \overline{Q}^G$ and $Q_{1jt} = \Phi_{jt}L^{\rho}_{1jt}$, it follows that

$$R^H_{1jt}(1-\epsilon)\frac{Q_{1jt}}{Q^H_{1jt}} - \frac{1+\theta}{\beta_L}B_{1jt} - X_{1jt} = 0. \tag{A.19}$$

Thus, combining equations (A.15) and (A.19), we have equation (13).

Lastly, it is interesting to consider if winning a procurement project will lead a firm to produce more for the private market (crowd-in) or less (crowd-out). To determine this, we evaluate the marginal profits of the winner when the total output is $\hat{Q}_{1jt} \equiv \overline{Q}^G + Q^H_{0jt}$; that is, $\hat{Q}_{1jt}$ is the hypothetical output of the firm in the $d = 1$ case such that there is neither crowd-in nor crowd-out. The winner would prefer to produce more (less) than $\hat{Q}_{1jt}$ if the marginal profit is positive (negative, respectively). Let the corresponding labor choice be $\hat{L}_{1jt}$ such that $\Phi_{jt}\hat{L}^{\rho}_{1jt} - \overline{Q}^G = Q^H_{0jt} = \Phi_{jt}L^{\rho}_{0jt}$. Note that, since $\rho > 1$ and $\hat{L}^{\rho}_{1jt} = L^{\rho}_{0jt} + \overline{Q}^G/\Phi_{jt}$, then $\hat{L}_{1jt} > L_{0jt}$. Evaluating equation (A.17) at $\hat{L}_{1jt}$, marginal profits for the firm if it wins and produces hypothetical output $\hat{Q}_{1jt}$ are,

$$\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=\hat{L}_{1jt}} = p_H\Phi_{jt}(1-\epsilon)\rho(Q^H_{0jt})^{-\epsilon}\hat{L}^{\rho-1}_{1jt} - \kappa_U U_{jt}(1+\theta)\hat{L}^{\theta}_{1jt} - \frac{p_M}{\beta_M}\Phi_{jt}\rho\hat{L}^{\rho-1}_{1jt}. \tag{A.20}$$

Multiplying by $L^{1-\rho}_{1jt}$ and substituting $Q^H_{0jt} = \Phi_{jt}L^{\rho}_{0jt}$, we have,

$$L^{1-\rho}_{1jt}\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=\hat{L}_{1jt}} = p_H\Phi^{1-\epsilon}_{jt}(1-\epsilon)\rho L^{-\rho\epsilon}_{0jt} - \kappa_U U_{jt}(1+\theta)\hat{L}^{\theta+1-\rho}_{1jt} - \frac{p_M}{\beta_M}\Phi_{jt}\rho. \tag{A.21}$$

Finally, substituting equation (A.14), this simplifies to,

$$L^{1-\rho}_{1jt}\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=\hat{L}_{1jt}} = \kappa_U U_{jt}(1+\theta)(L^{\theta+1-\rho}_{0jt} - \hat{L}^{\theta+1-\rho}_{1jt}). \tag{A.22}$$

Since $\hat{L}_{1jt} > L_{0jt}$, we have that $\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=\hat{L}_{1jt}} < 0$ if $\theta + 1 - \rho > 0$, and $\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=\hat{L}_{1jt}} > 0$ otherwise. Therefore, winning a government project crowds-out private projects when $1 + \theta > \rho$ and crowds-in if $1 + \theta < \rho$.

3. Worker Rents Expressions

We derive the key expressions for worker rents in Section IE. First, following Lamadon, Mogstad and Setzler (2022), total worker rents at firm j in year t are,

$$V_{jt}(W_{jt}) = \int_0^{W_{jt}} (W_{jt} - W)\frac{dL_{jt}(W)}{dW}dW. \tag{A.23}$$

Define $\omega \equiv \frac{W}{W_{jt}}$ so that $\frac{d\omega}{dW} = \frac{1}{W_{jt}}$, and note that labor supply can be expressed in terms ω as $\tilde{L}(\omega W_{jt}) = \omega^{1/\theta} L(W_{jt})$. Thus, $\frac{dL_{jt}(W)}{dW} dW = \frac{d\tilde{L}_{jt}(\omega W_{jt})}{d\omega} d\omega$. Moreover, $\frac{d\tilde{L}_{jt}(\omega W_{jt})}{d\omega} = L_{jt}(W_{jt}) \frac{\partial \omega^{1/\theta}}{\partial \omega}$, which implies

$$V_{jt}(W_{jt}) = W_{jt} \int_0^1 (1-\omega) \frac{d\tilde{L}(\omega W_{jt})}{d\omega} d\omega = W_{jt} L_{jt}(W_{jt}) \int_0^1 (1-\omega) \frac{\partial \omega^{1/\theta}}{\partial \omega} d\omega = \frac{W_{jt} L_{jt}(W_{jt})}{1+1/\theta}$$

Second, we derive the decomposition of incidence for incumbents and new hires. Let $\mathcal{I}_{jt}$ denote the set of incumbent workers at firm j. For two potential wages W_{1jt} and W_{0jt}, the corresponding rents V_{1ijt} and V_{0ijt} for any $i \in \mathcal{I}_{jt}$ must satisfy,

$$\mathcal{U}_{it}(j, W_{1jt} - V_{1ijt}) = \max_{j' \neq j} \mathcal{U}_{it}(j', W_{j',t}) \quad \text{and} \quad \mathcal{U}_{it}(j, W_{0jt} - V_{0ijt}) = \max_{j' \neq j} \mathcal{U}_{it}(j', W_{j',t}).$$

Since the right-hand side is the same in both of these equations (that is, the outside option is unchanged by a wage increase at the incumbent employer), it follows that $\mathcal{U}_{it}(j, W_{1jt} - V_{1ijt}) = \mathcal{U}_{it}(j, W_{0jt} - V_{0ijt}),\ \forall i \in \mathcal{I}_{jt}$, which can only be satisfied by $V_{1ijt} - V_{0ijt} = W_{1jt} - W_{0jt},\ \forall i \in \mathcal{I}_{jt}$. Thus, the incidence for incumbents is

$$\underbrace{\sum_{\mathcal{I}_{jt}} (V_{1ijt} - V_{0ijt})}_{\text{Incidence for incumbents}} = \underbrace{L_{0jt}}_{\text{Number of incumbents}} \times \underbrace{(W_{1jt} - W_{0jt})}_{\text{Incidence for each incumbent}}.$$

The incidence for new hires is then,

$$\underbrace{V_{1jt} - V_{0jt}}_{\text{Incidence}} - \underbrace{L_{0jt}(W_{1jt} - W_{0jt})}_{\text{Incidence for incumbents}} = \underbrace{\frac{W_{1jt} L_{1jt}}{1+1/\theta} - \frac{W_{0jt} L_{0jt}}{1+1/\theta} - L_{0jt}(W_{1jt} - W_{0jt})}_{\text{Incidence for new hires}}$$

which can be rearranged as the decomposition in Section IE.

4. Over-identifying Restriction

We now derive equation (26). Taking the log of both sides of equation (13) for the $d = 1$ case, we have,

$$\log(1-\epsilon) + r^H_{1jt} + q_{1jt} - q^H_{1jt} = \log\left(\frac{1+\theta}{\beta_L} B_{1jt} + X_{1jt}\right).$$

From equation (7), $r^H_{1jt} = \log p_H + (1-\epsilon) q^H_{1jt}$, so $q^H_{1jt} = \frac{1}{1-\epsilon} r^H_{1jt} - \frac{1}{1-\epsilon} \log p_H$. From equation (10), $q_{1jt} = \rho \ell_{1jt} + \phi_{jt} + e_{jt}$. Substituting, we have

$$\log(1-\epsilon) + r^H_{1jt} + (\rho \ell_{1jt} + \phi_{jt} + e_{jt}) - \left(\frac{1}{1-\epsilon} r^H_{1jt} - \frac{1}{1-\epsilon} \log p_H\right) = \log\left(\frac{1+\theta}{\beta_L} B_{1jt} + X_{1jt}\right),$$

which can be rearranged as equation (26).

B. Product Market with Perfect Competition

We now solve the firm's problem in the private product market assuming the firm is a price-taker ($\epsilon = 0$). Denote the competitive price as p_H. In terms of the composite production function $Q_{jt} = \Phi_{jt} L_{jt}^{\rho}$, the firm's problem is

$$\max_{L_{jt}:\, \Phi_{jt} L_{jt}^{\rho} \geq d\overline{Q}^G} \quad p_H \left(\Phi_{jt} L_{jt}^{\rho} - d\overline{Q}^G \right) - \kappa_U U_{jt} L_{jt}^{1+\theta} - \frac{p_M}{\beta_M} \Phi_{jt} L_{jt}^{\rho}$$

where the government's output must be produced if the firm receives a procurement contract ($\Phi_{jt} L_{jt}^{\rho} \geq d\overline{Q}^G$). We consider three cases:

Suppose $d = 0$. The government constraint is always satisfied, so we can ignore this constraint. The profit-maximizing solution is simply $Q_{0jt} = \Phi_{jt} L_{0jt}^{\rho}$ and

$$L_{0jt} = \left(\left(p_H - \frac{p_M}{\beta_M} \right) \frac{\Phi_{jt} \rho}{\kappa_U U_{jt} (1+\theta)} \right)^{\frac{1}{\theta+1-\rho}}.$$

Suppose $d = 1$ and $Q_{0jt} > \overline{Q}^G$. Then, the solution $L_{1jt}^{interior} = L_{0jt}$ and $Q_{1jt}^{interior} = Q_{0jt}$ satisfies the government constraint and otherwise solves the profit-maximization problem, so this is the optimal solution. An implication is that $Q_{1jt}^{interior}$ is invariant to marginal changes in the size of the government contract, i.e., government projects crowd-out the firm's private market production one-for-one. Since input costs are not affected by receiving a procurement contract, the opportunity cost of receiving a procurement contract is simply the loss in revenues in the private product market, $\sigma_{jt}^{interior} = p_H \left(Q_{0jt} - \left(Q_{1jt}^{interior} - \overline{Q}^G \right) \right) = p_H \overline{Q}^G$.

Suppose $d = 1$ and $Q_{0jt} \leq \overline{Q}^G$. Then, the firm is at the corner solution in which it only produces for the government market, i.e., $Q_{1jt}^{corner} = \overline{Q}^G$ and $L_{1jt}^{corner} = \left(\overline{Q}^G / \Phi_{jt} \right)^{1/\rho}$. The opportunity cost is $\sigma_{jt}^{corner} = p_H Q_{0jt} - \left\{ T_{jt}\left(L_{0jt}\right) - T_{jt}\left(L_{1jt}^{corner}\right) \right\}$, where $T_{jt}(L) \equiv \kappa_U U_{jt} L^{1+\theta} + \frac{p_M}{\beta_M} \Phi_{jt} L^{\rho}$ is the total cost of production using labor L.

C. Cobb-Douglas Production Function

1. *Cobb-Douglas Model: Composite Production Function*

Consider a Cobb-Douglas production function (in physical units)

$$Q_{jt} = \Omega_{jt} L_{jt}^{\beta_L} K_{jt}^{\beta_K} M_{jt}^{\beta_M}. \tag{A.24}$$

Given any production level Q_{jt}, the firm can find the most cost efficient combination (L_{jt}, K_{jt}, M_{jt}) by solving the cost-minimization problem,

$$\min_{L_{jt},K_{jt},M_{jt}} \quad C_{jt} \quad \text{s.t.} \quad Q_{jt} = \Omega_{jt} L_{jt}^{\beta_L} K_{jt}^{\beta_K} M_{jt}^{\beta_M}. \tag{A.25}$$

where $C_{jt} \equiv U_{jt} L_{jt}^{1+\theta} + p_K K_{jt} + p_M M_{jt}$ denotes the total cost. This leads to the Lagrangian,

$$\mathcal{L}_{jt} \equiv U_{jt} L_{jt}^{1+\theta} + p_K K_{jt} + p_M M_{jt} + \lambda_{jt}(Q_{jt} - \Omega_{jt} K_{jt}^{\beta_K} L_{jt}^{\beta_L} M_{jt}^{\beta_M}) \tag{A.26}$$

where λ_{jt} is the Lagrange multiplier. The first-order conditions are:

$$\begin{aligned} p_K &= \lambda_{jt}\Omega_{jt}\beta_K K_{jt}^{\beta_K - 1} L_{jt}^{\beta_L} M_{jt}^{\beta_M}, \\ (1+\theta) U_{jt} L_{jt}^{\theta} &= \lambda_{jt}\Omega_{jt}\beta_L K_{jt}^{\beta_K} L_{jt}^{\beta_L - 1} M_{jt}^{\beta_M}, \\ p_M &= \lambda_{jt}\Omega_{jt}\beta_M K_{jt}^{\beta_K} L_{jt}^{\beta_L} M_{jt}^{\beta_M - 1}. \end{aligned} \tag{A.27}$$

We can use these first-order conditions to write the optimal choices of capital and intermediate inputs as a function of labor

$$K_{jt} = \frac{\beta_K}{\beta_L} \frac{(1+\theta)U_{jt}}{p_K} L_{jt}^{1+\theta} = \chi^{(K)} U_{jt} L_{jt}^{1+\theta} \text{ and } M_{jt} = \frac{\beta_M}{\beta_L} \frac{(1+\theta)U_{jt}}{p_M} L_{jt}^{1+\theta} = \chi^{(M)} U_{jt} L_{jt}^{1+\theta} \tag{A.28}$$

where $\chi^{(K)} \equiv \frac{\beta_K}{\beta_L}\frac{(1+\theta)}{p_K}$ and $\chi^{(M)} \equiv \frac{\beta_M}{\beta_L}\frac{(1+\theta)}{p_M}$. We can substitute these expressions into $Q_{jt} = \Omega_{jt} L_{jt}^{\beta_L} K_{jt}^{\beta_K} M_{jt}^{\beta_M}$ and obtain

$$Q_{jt} = \Omega_{jt} \left[\chi_j^{(K)} U_{jt} L_{jt}^{1+\theta}\right]^{\beta_K} L_j^{\beta_L} \left[\chi_j^{(M)} U_{jt} L_{jt}^{1+\theta}\right]^{\beta_M} = \Phi_{jt} L_{jt}^{\rho} \tag{A.29}$$

where $\Phi_{jt} \equiv \Omega_{jt}\left[\chi^{(K)} U_{jt}\right]^{\beta_K}\left[\chi^{(M)} U_{jt}\right]^{\beta_M}$ and $\rho \equiv \beta_L + (1+\theta)(\beta_K + \beta_M)$. We can also use equations (A.29) and equation (A.28) to rewrite the firm's problem in the private product market as

$$\max_{L_{djt}} \pi_{djt} = p_H(\Phi_{jt} L_{djt}^{\rho} - \bar{Q}^G d)^{1-\varepsilon} - \chi^{(W)} U_{jt} L_{djt}^{1+\theta}, \tag{A.30}$$

where cost-minimization implies $C_{djt} = \chi^{(W)} U_{jt} L_{djt}^{\theta+1}$, $\chi^{(W)} \equiv \frac{\rho}{\beta_L} \equiv \left(1 + \frac{(\beta_M+\beta_K)(1+\theta)}{\beta_L}\right)$.

2. Cobb-Douglas Model: First-order Conditions

We now derive the profit-maximizing first-order conditions in the model with Cobb-Douglas production. These derivations assume $\rho \equiv \beta_L + (1+\theta)(\beta_K + \beta_M) > 1$ and $\varepsilon > 0$.

If the firm loses the auction, its profit maximization problem is

$$\max_{L_{0jt}} p_H(\Phi_{jt}L_{0jt}^{\rho})^{1-\varepsilon} - \chi^{(W)}U_{jt}L_{0jt}^{1+\theta}. \tag{A.31}$$

The first-order condition is,

$$\rho(1-\varepsilon)p_H\Phi_{jt}^{1-\varepsilon}L_{0jt}^{\rho(1-\varepsilon)-1} = \chi^{(W)}U_{jt}L_{0jt}^{\theta}(1+\theta), \tag{A.32}$$

which implies,

$$L_{0jt} = \left[\frac{\rho(1-\varepsilon)p_H\Phi_{jt}^{1-\varepsilon}}{\chi^{(W)}U_{jt}(1+\theta)}\right]^{\frac{1}{\theta+1-\rho(1-\varepsilon)}}. \tag{A.33}$$

Thus $0 < L_{0jt} < \infty$.

Similarly, if the firm wins the auction, the profit maximization problem is:

$$\max_{L_{1jt}:\, \Phi_{jt}L_{1jt}^{\rho} \geq \bar{Q}^G} p_H(\Phi_{jt}L_{1jt}^{\rho} - \bar{Q}^G)^{1-\varepsilon} - \chi^{(W)}U_{jt}L_{1jt}^{1+\theta}. \tag{A.34}$$

The first-order condition is

$$\frac{\partial \pi_{1jt}}{\partial L_{1jt}} \equiv \rho(1-\varepsilon)\Phi_{jt}p_H(\Phi_{jt}L_{1jt}^{\rho} - \bar{Q}^G)^{-\varepsilon}L_{1jt}^{\rho-1} - \chi^{(W)}U_{jt}L_{1jt}^{\theta}(1+\theta) = 0, \tag{A.35}$$

which implies,

$$\rho(1-\varepsilon)\Phi_{jt}p_H(\Phi_{jt}L_{1jt}^{\rho} - \bar{Q}^G)^{-\varepsilon} = \chi^{(W)}U_{jt}L_{1jt}^{1+\theta-\rho}(1+\theta). \tag{A.36}$$

As $\Phi_{jt}L_{1jt}^{\rho}$ approaches $\bar{Q}^G$, the left-hand side of equation (A.36) approaches infinity while the RHS approaches a constant. Thus, $\Phi_{jt}L_{1jt}^{\rho} > \bar{Q}^G$ is necessary to satisfy the equation. Since $Q_{1jt} = \Phi_{jt}L_{1jt}^{\rho}$, it follows that $Q_{1jt}^H = Q_{1jt} - \bar{Q}^G > 0$, so the winning firm always produces for the private market.

Furthermore, since the solution is interior (i.e., $L_{0jt} \geq \overline{Q}^G$) due to $\epsilon > 0$, equation (A.32) implies $\rho(1-\varepsilon)\Phi_{jt}p_H(\Phi_{jt}L_{0jt}^{\rho})^{-\varepsilon}L_{0jt}^{\rho-1} - \chi^{(W)}U_{jt}L_{0jt}^{\theta}(1+\theta) = 0$ and therefore $\rho(1-\varepsilon)\Phi_{jt}p_H(\Phi_{jt}L_{0jt}^{\rho} - \bar{Q}^G)^{-\varepsilon}L_{0jt}^{\rho-1} - \chi^{(W)}U_{jt}L_{0jt}^{\theta}(1+\theta) \equiv \frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=L_{0jt}} > 0$. Thus, $\frac{\partial \pi_{1jt}}{\partial L_{1jt}}|_{L_{1jt}=L_{0jt}} > 0$, so total production will be larger if the firm receives a procurement contract than if it does not.

3. *Cobb-Douglas Model: Identification*

We now show identification of $(1-\epsilon, \rho, \beta_L)$ in the model with a Cobb-Douglas production function.

In the $d = 0$ case, revenues are related to labor by

$$r_{jt} = \log p_H + (1-\epsilon)\,\phi_{jt} + \rho\,(1-\epsilon)\,\ell_{jt} \tag{A.37}$$

From this, we can identify $\rho\,(1-\epsilon)$ by regressing r_{jt} on ℓ_{jt} controlling for ϕ_{jt} among $D_{jt} = 0$ firms. In practice, we can control for $(\widehat{u}_{jt}, Z_{jt})$ in place of ϕ_{jt} as in equation (23) due to the invertibility of bids with respect to TFP, conditional on amenities. Thus, $\rho\,(1-\epsilon)$ is recovered by the estimator

$$\widehat{\rho\,(1-\epsilon)} \equiv \frac{\mathrm{Cov}\left[r_{jt}, \ell_{jt} | \widehat{u}_{jt}, Z_{jt}, D_{jt} = 0\right]}{\mathrm{Var}\left[\ell_{jt} | \widehat{u}_{jt}, Z_{jt}, D_{jt} = 0\right]}. \tag{A.38}$$

In the $d = 0$ case, equation (A.32) implies

$$\rho\,(1-\epsilon)\,\frac{R^H_{0jt}}{L_{0jt}} = \chi^{(W)} U_{jt} L^{\theta}_{0jt} (1+\theta).$$

Since we showed above that cost-minimization requires $C_{djt} = \chi^{(W)} U_{jt} L^{\theta+1}_{djt}$, it follows that

$$\rho\,(1-\epsilon) = (1+\theta)\,\frac{C_{0jt}}{R^H_{0jt}}. \tag{A.39}$$

Taking expectations in logs and rearranging, this yields another estimator that over-identifies $\rho\,(1-\epsilon)$:

$$\widetilde{\rho\,(1-\epsilon)} \equiv \exp\left(\log(1+\theta) + \mathbb{E}\left[c_{jt} - r^H_{jt} | D_{jt} = 0\right]\right). \tag{A.40}$$

In the $d = 1$ case, multiplying both sides of equation (A.35) by L_{1jt} implies

$$\rho(1-\varepsilon)\Phi_{jt} p_H (\Phi_{jt} L^{\rho}_{1jt} - \bar{Q}^G)^{-\varepsilon} L^{\rho}_{1jt} = \chi^{(W)} U_{jt} L^{\theta+1}_{1jt} (1+\theta) = (1+\theta) C_{jt}$$

Furthermore, since $(\Phi_{jt} L^{\rho}_{1jt} - \bar{Q}^G)^{-\varepsilon} = (Q^H_{1jt})^{-\epsilon} = (R^H_{1jt}/p_H)^{\frac{-\epsilon}{1-\epsilon}}$, we can rewrite this expression as

$$\rho(1-\varepsilon) p_H (R^H_{1jt}/p_H)^{\frac{-\epsilon}{1-\epsilon}} \Phi_{jt} L^{\rho}_{1jt} = (1+\theta) C_{jt}$$

Taking logs,

$$\log\rho + \log(1-\varepsilon) + \log p_H + \frac{-\epsilon}{1-\epsilon} r^H_{1jt} - \frac{-\epsilon}{1-\epsilon}\log p_H + \phi_{jt} + \rho\ell_{1jt} = \log(1+\theta) + c_{jt}$$

Rearranging, this gives,

$$\underbrace{c_{jt} + \frac{\epsilon}{1-\epsilon} r_{jt}^{H}}_{\Lambda_{jt}^{\mathrm{CD}}(\epsilon)} = \text{constant} + \phi_{jt} + \rho \ell_{jt}, \tag{A.41}$$

where constant $\equiv \log \rho + \log(1-\varepsilon) + \frac{1}{1-\epsilon} \log p_H - \log(1+\theta)$. Thus, for any candidate value of ϵ, a regression of $\Lambda_{jt}^{\mathrm{CD}}(\epsilon)$ on ℓ_{jt} controlling for ϕ_{jt} for the winners identifies ρ. Since $\rho\,(1-\epsilon)$ is identified above, this implies $(1-\epsilon)$ is uniquely determined by this implicit system of equations.

Furthermore, since we showed above that cost-minimization requires $C_{jt} = \frac{\rho}{\beta_L} B_{jt}$, the expected labor share of costs is

$$\frac{\beta_L}{\rho} = \mathbb{E}\left[\frac{B_{jt}}{C_{jt}}\right], \tag{A.42}$$

so we identify β_L given ρ.

In practice, we simultaneously estimate $(1-\epsilon, \rho, \beta_L)$ by applying equally-weighted GMM to equations (A.38), (A.40), (A.41), and (A.42).

For the remaining parameters, note that $X_{jt} = \frac{(1+\theta)\beta_M}{\beta_L} B_{jt}$ and $p_K K_{jt} = \frac{(1+\theta)\beta_K}{\beta_L} B_{jt}$, which implies the following expressions:

$$\beta_M = \exp\left(\mathbb{E}\left[x_{jt} - b_{jt}\right] - \log \frac{(1+\theta)}{\beta_L}\right), \tag{A.43}$$

$$\beta_K = \exp\left(\mathbb{E}\left[\log\left(p_K K_{jt}\right) - b_{jt}\right] - \log \frac{(1+\theta)}{\beta_L}\right), \tag{A.44}$$

$$\mathbb{E}\left[u_{jt}\right] = \mathbb{E}\left[b_{jt}\right] - (1+\theta)\,\mathbb{E}\left[\ell_{jt}\right], \tag{A.45}$$

$$\log p_H = \mathbb{E}\left[r_{jt}\right] - \rho\,(1-\epsilon)\,\mathbb{E}\left[\ell_{jt}\right], \tag{A.46}$$

where we normalize $\mathbb{E}[\phi_{jt}] = 0$ without loss of generality.

D. Expected Impacts of an Increase in Market Power

1. Mathematical Representation of the Expected Impacts of an Increase in Market Power

Set up:

For simplicity, we consider a production function in which labor is the only input, returns to scale are constant ($\rho = 1$), and firms can only sell output to the private market when deriving theoretical predictions. We focus on firm j at time t, omitting these subscripts without loss of generality, and normalize TFP as $\Phi = 1$. The production function is then $Q = L$. This implies that revenue can be expressed in terms of labor as $R = p_H L^{1-\epsilon}$, so marginal revenue is $\text{MRP} = p_H(1-\epsilon)L^{-\epsilon}$. Since labor is the only input, the marginal cost of production is given by the marginal cost of labor, which is $\text{MCL} = U\,(1+\theta)\,L^{\theta}$. We solve for the baseline equilibrium by equating MRP and MCL. The baseline equilibrium is characterized by $\overline{L} = \overline{Q} = \left(\frac{p_H}{U}\frac{1-\epsilon}{1+\theta}\right)^{\frac{1}{\theta+\epsilon}}$, $\overline{P} = p_H \overline{L}^{-\epsilon}$, $\overline{R} = p_H \overline{L}^{1-\epsilon}$, and $\overline{W}_{jt} = U\overline{L}^{\theta}$.

Rotation of labor supply curve:

We now consider a compensated rotation of the labor supply curve. In particular, consider an (inverse) labor supply curve $W\,(L|U', \theta') = U' L^{\theta'}$ for some $\theta' \neq \theta$. This labor supply curve is a "rotation" around the initial equilibrium only if $W\left(\overline{L}|U', \theta'\right) = \overline{W}$; that is, the baseline labor quantity receives the same wage after the rotation as it did in the baseline equilibrium. This rotation $W\left(\overline{L}|U', \theta'\right) = \overline{W}$ is solved by $U' = \overline{W}\,\overline{L}^{\,-\theta'}$; that is, there is a unique "compensation" $U' - U$ to the location parameter of the labor supply curve such that $W\,(L|U', \theta')$ is a "rotation" around the initial equilibrium and $\theta' \neq \theta$.

Suppose labor supply is rotated to become more inelastic; that is, $\theta' > \theta$, which also implies $U' < U$. The new equilibrium satisfies $L' = Q' = \left(\frac{p_H}{U'}\frac{1-\epsilon}{1+\theta'}\right)^{\frac{1}{\theta'+\epsilon}} = \overline{L}\left(\frac{1+\theta}{1+\theta'}\right)^{\frac{1}{\theta'+\epsilon}}$. Since $\left(\frac{1+\theta}{1+\theta'}\right)^{\frac{1}{\theta'+\epsilon}} < 1$, then $L' < \overline{L}$ and thereby $Q' < \overline{Q}$. An implication is that $p_H\,(Q')^{-\epsilon} > p_H\,(Q)^{-\epsilon}$, so $P' > \overline{P}$. Another implication is that $W' = U'\,(L')^{\theta'} = U'\left(\overline{L}\left(\frac{1+\theta}{1+\theta'}\right)^{\frac{1}{\theta'+\epsilon}}\right)^{\theta'} = \overline{W}\frac{U'}{U}\left(\frac{1+\theta}{1+\theta'}\right)^{\frac{\theta'}{\theta'+\epsilon}}$. Since $\frac{U'}{U}\left(\frac{1+\theta}{1+\theta'}\right)^{\frac{\theta'}{\theta'+\epsilon}} < 1$, it follows that $W' < \overline{W}$. Therefore, a compensated rotation of the labor supply curve to become less elastic results in reductions in the firm's employment, wage, and output, as well as an increase in its price.

Rotation of product demand curve:

We now consider a compensated rotation of the product demand curve. In particular, consider an (inverse) product demand curve $P\,(Q|p'_H, \epsilon') = p'_H Q^{-\epsilon'}$ for some $\epsilon' \neq \epsilon$. This product demand curve is a "rotation" around the initial equilibrium only if $P\left(\overline{Q}|p'_H, \epsilon'\right) = \overline{P}$; that is, the baseline output quantity receives the same price after the rotation as it did in the baseline equilibrium. This rotation $P\left(\overline{Q}|p'_H, \epsilon'\right) = \overline{P}$ is solved by $p'_H = \overline{P}\overline{Q}^{\epsilon'}$; that is, there is a unique "compensation" $p'_H - p_H$ to the location parameter of the product demand curve such that $P\,(Q|p'_H, \epsilon')$ is a "rotation" around the initial equilibrium and $\epsilon' \neq \epsilon$.

Suppose product demand is rotated to become more inelastic; that is, $\epsilon' > \epsilon$, which also implies $p'_H > p_H$. The new equilibrium satisfies $L' = Q' = \left(\frac{p'_H}{U}\frac{1-\epsilon'}{1+\theta}\right)^{\frac{1}{\theta+\epsilon'}} = \overline{L}\left(\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{1}{\theta+\epsilon'}}$. Since $\left(\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{1}{\theta+\epsilon'}} < 1$, then $L' < \overline{L}$ and thereby

$Q' < \overline{Q}$. An implication is that $U(L')^{\theta} < U(\overline{L})^{\theta}$, so $W' < \overline{W}$. Another implication is that $P' = p'_H (Q')^{-\epsilon'} = p'_H \left(\overline{Q}\left(\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{1}{\theta+\epsilon'}}\right)^{-\epsilon'} = \overline{P}\frac{p'_H}{p_H}\left(\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{-\epsilon'}{\theta+\epsilon'}}$. Since $\frac{p'_H}{p_H}\left(\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{-\epsilon'}{\theta+\epsilon'}} > 1$, it follows that $P' > \overline{P}$. Therefore, a compensated rotation of the product demand curve to become less elastic results in reductions in the firm's employment, wage, and output, as well as an increase in its price.

Rotation of both labor supply and product demand curves:

Lastly, we consider rotating both the labor supply and product demand curves to become more inelastic; that is, $\epsilon' > \epsilon$ and $\theta' > \theta$. Following the same logic as above, $L' = \overline{L}\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{1}{\theta'+\epsilon'}}$. Since $\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{1}{\theta'+\epsilon'}} < 1$, then $L' < \overline{L}$ and thereby $Q' < \overline{Q}$. Since $W' = \overline{W}\frac{U'}{U}\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{\theta'}{\theta'+\epsilon'}}$ and $\frac{U'}{U}\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{\theta'}{\theta'+\epsilon'}} < 1$, it follows that $W' < \overline{W}$. Since $P' = \overline{P}\frac{p'_H}{p_H}\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{-\epsilon'}{\theta'+\epsilon'}}$ and $\frac{p'_H}{p_H}\left(\frac{1+\theta}{1+\theta'}\frac{1-\epsilon'}{1-\epsilon}\right)^{\frac{-\epsilon'}{\theta'+\epsilon'}} > 1$, it follows that $P' > \overline{P}$. Therefore, a simultaneous compensated rotation of both the labor supply and product demand curves to become less elastic results in reductions in the firm's employment, wage, and output, as well as an increase in its price.

Lastly, we show that the impacts of increased market power in one market are attenuated by the existence of market power in the other market. In particular, we show that

$$\left.\frac{\partial^2 L}{\partial\theta'\partial\epsilon'}\right|_{\left\{P\left(\overline{Q}|p'_H,\epsilon'\right)=\overline{P},\, W\left(\overline{L}|U',\theta'\right)=\overline{W}\right\}} = \overline{L}\frac{1}{(\theta+\epsilon)^2}\left[\frac{1}{1+\theta}+\frac{1}{1-\epsilon}+\frac{1}{(1+\theta)(1-\epsilon)}\right] > 0.$$

We start with $L = \overline{L}\left[\frac{(1+\theta)(1-\epsilon')}{(1-\epsilon)(1+\theta')}\right]^{\frac{1}{\theta'+\epsilon'}}$, which implies

$$\log L - \log\overline{L} = \frac{1}{\theta'+\epsilon'}\left[\log\frac{(1+\theta)(1-\epsilon')}{(1-\epsilon)(1+\theta')}\right]$$

Setting $\theta = \theta'$ and $\epsilon = \epsilon'$ delivers $\log L - \log\overline{L} = 0$. We can calculate the following derivatives:

$$\frac{d\log L}{d\theta'} = \frac{1}{L}\frac{dL}{d\theta'} = -\frac{1}{(\theta'+\epsilon')^2}\left[\log\frac{(1+\theta)(1-\epsilon')}{(1-\epsilon)(1+\theta')}\right] - \frac{1}{\theta'+\epsilon'}\frac{1}{1+\theta'}$$

$$\frac{d\log L}{d\epsilon'} = \frac{1}{L}\frac{dL}{d\epsilon'} = -\frac{1}{(\theta'+\epsilon')^2}\left[\log\frac{(1+\theta)(1-\epsilon')}{(1-\epsilon)(1+\theta')}\right] - \frac{1}{\theta'+\epsilon'}\frac{1}{1-\epsilon'}$$

Substituting,

$$\frac{dL}{d\theta'} = -\left[\frac{1}{(\theta'+\epsilon')^2}\left[\log\frac{(1+\theta)(1-\epsilon')}{(1-\epsilon)(1+\theta')}\right] + \frac{1}{\theta'+\epsilon'}\frac{1}{1+\theta'}\right] L$$

$$\frac{dL}{d\theta'} = -\frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L$$

$$\frac{dL}{d\epsilon'} = -\frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1-\epsilon'}\right] L$$

Thus,

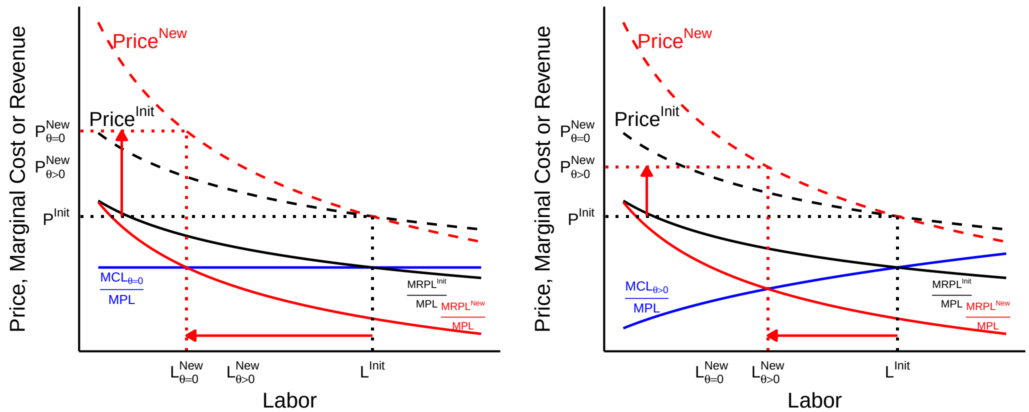
$$\begin{aligned}\frac{d^2L}{d\theta' d\epsilon'} &= \frac{1}{(\theta'+\epsilon')^2}\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L - \frac{1}{\theta'+\epsilon'} L \frac{d\log L}{d\epsilon'} - \frac{dL}{d\epsilon'}\frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] \\ &= \frac{1}{(\theta'+\epsilon')^2}\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L + \frac{1}{\theta'+\epsilon'}\left[\frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1-\epsilon'}\right]\right] L \\ &\quad + \frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1-\epsilon'}\right]\frac{1}{\theta'+\epsilon'}\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L \\ &= \frac{1}{(\theta'+\epsilon')^2}\Bigg(\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L + \left[\log L - \log\overline{L} + \frac{1}{1-\epsilon'}\right] L \\ &\quad + \left[\log L - \log\overline{L} + \frac{1}{1-\epsilon'}\right]\left[\log L - \log\overline{L} + \frac{1}{1+\theta'}\right] L\Bigg)\end{aligned}$$

Finally, evaluating at $L = \overline{L}$, $\theta = \theta'$, and $\epsilon = \epsilon'$ delivers:

$$\frac{d^2L}{d\theta' d\epsilon'} = \overline{L}\frac{1}{(\theta'+\epsilon')^2}\left[\frac{1}{1+\theta'} + \frac{1}{1-\epsilon'} + \frac{1}{1-\epsilon'}\frac{1}{1+\theta'}\right] > 0.$$

2. Graphical Representation of the Expected Impacts of an Increase in Product Market Power

In Figures A.1a-A.1b, we use the compensated rotation to show the expected impact of an increase in product market power and how it depends on the presence or absence of labor market power. The initial inverse product demand elasticity is $\epsilon^{\text{Init}} > 0$, initial employment is L^{Init}, and initial price is P^{Init}. In Figure A.1a, labor supply is perfectly elastic ($\theta = 0$). When the inverse product demand curve rotates from the initial curve labeled $\text{Price}^{\text{Init}}$ (corresponding to ϵ^{Init}) to the less elastic curve labeled $\text{Price}^{\text{New}}$ (corresponding to ϵ^{New}), the corresponding productivity-adjusted MRPL curve falls from the initial curve labeled $\frac{\text{MRPL}^{\text{Init}}}{\text{MPL}}$ to the new curve labeled $\frac{\text{MRPL}^{\text{New}}}{\text{MPL}}$ through the relationship $\frac{\text{MRPL}}{\text{MPL}} = (1-\epsilon) \times \text{Price}$. Since the first-order condition equates MRPL and MCL, and MRPL is lower for



(a) Change in Product Market Power without Labor Market Power

(b) Change in Product Market Power with Labor Market Power

Figure A.1. : Expected Impacts of Changes in Market Power

Notes: This figure presents the expected impacts of increasing product market power. For simplicity, we parameterize the production function as $Q_{jt} = \Phi_{jt} L_{jt}$, although the results in the text hold for more general production functions. We consider the compensated rotation of the Price curve (i.e. the inverse product demand curve) around the initial choices $(L^{\text{Init}}, P^{\text{Init}})$. We denote the productivity-adjusted MCL curve (in blue) by $\frac{\text{MCL}_{\theta>0}}{\text{MPL}}$ for the case with labor market power and $\frac{\text{MCL}_{\theta=0}}{\text{MPL}}$ for the case without labor market power.

all values of labor as product market power increases, it follows that the firm finds it profitable to reduce labor. Since the price curve is downward sloping, a reduction in labor raises the price.

In Figure A.1b, we perform exactly the same analysis as in Figure A.1a, except that the firm now has labor market power. To introduce labor market power, we rotate the labor supply curve by increasing θ from zero to a positive value, and then change the location parameter U such that the initial optimum is identical to Figure A.1a. Next, we rotate the product demand curve to increase product market power. The expected impact of increased product market power is qualitatively similar to Figure A.1a: the MRPL falls at all values of labor, so MRPL and MCL are equalized by a smaller choice of labor and higher prices. However, in contrast to Figure A.1a, the wage falls as output declines in Figure A.1b due to $\theta > 0$, so the MCL is lower at the new optimal choice of labor in Figure A.1b.

While the *qualitative* effects of increased product market power are similar in Figures A.1a and A.1b, the *magnitude* of the expected decrease in labor and increase in price relative to the initial optimum is stronger in Figure A.1a due to the absence of labor market power. The intuition for this result is straightforward: if the firm experiences an increase in product market power, it wants to increase the price it charges by reducing output and, thereby, employment. However, the firm will choose to reduce output and employment less if it is facing upward-sloping labor supply ($\theta > 0$), as lower employment decreases the wage it has to pay and, thus, the marginal cost of labor. By contrast, in Figure A.1a, the wage remains constant as employment falls, giving the firm no labor market power to exploit, and a greater reduction in output is required in order for MRPL and MCL to be equalized, leading to a greater increase in prices.

E. Additional Institutional Details on the Construction Industry and Procurement Auctions

1. *Prevalence of Non-wage Compensation*

The Economic Census of the Construction Sector (EC), which is collected every five years by the US Census Bureau, provides informative descriptive statistics on the wage and non-wage compensation paid by the US construction industry (United States Census Bureau, 2023). The EC provides measures of non-wage compensation separately for legally-required fringe benefits (social security, unemployment insurance, worker injury-compensation insurance, etc.) and voluntary fringe benefits (health insurance, pensions, training, etc.), as well as total payroll. We analyze the EC data for the construction industry, aggregated to the state-level at five-year frequency, for the period from 1977 to 2017. Dollar values are adjusted for inflation (United States Bureau of Labor Statistics, 2023).

In Figure A.12, we present the share of total compensation from wages, legally-required fringe benefits, and voluntary fringe benefits over time. In 2012, which is near the end of the time frame considered in our main analysis, we find that about 10% of total compensation is due to legally-required benefits and about 10% is due to voluntary benefits, with wages accounting for the remaining 80%. Thus, voluntary benefits – which is the component of non-wage compensation that the firm can in principle adjust – accounts for only one-tenth of total compensation in the construction industry.

It is perhaps not surprising that the construction industry primarily compensates workers through wages rather than voluntary benefits, as these benefits may be costly to provide. Industry experts summarize the role of wages versus non-wage benefits in recruiting workers to construction jobs by noting that "base pay has been the single most important piece of compensation" and "benefits aren't usually a driving factor in recruiting."[1] However, some workers may be offered non-wage benefits that are proportional to wages, such as bonuses and incentives which are offered as a percentage of base salary and thus adjust when wages increase.[2] We show in Supplemental Appendix H that proportional adjustments in non-wage benefits do not introduce bias in our estimation of the labor supply curve.

[1]See "What you need to know about compensation in construction" by Carpenter-Beck, https://www.sage.com/en-us/blog/need-know-compensation-construction/.

[2]See "Competitive pay to recruit and retain employees" by Robinson, https://www.sage.com/en-us/blog/competitive-pay-to-recruit-and-retain-employees/.

2. Relevance of Prevailing Wage Laws

Prevailing wage laws require that workers employed by private construction firms on government-funded construction projects be paid at least the wages and benefits paid to similar workers in the same location where the project is located. The Davis-Bacon Act was passed by Congress in 1931 to require that private construction firms pay the prevailing wage to their employees on all federally-funded construction projects. Subsequently, most state governments passed so-called "little Davis-Bacon" laws to extend prevailing wage requirements to state-funded construction projects. However, 15 of those states have since repealed their prevailing wage laws.[3]

One potential concern in our estimation of the labor supply elasticity is that first-time procurement auction winners may become subject to the prevailing wage for the first time, which could force them to increase the wages of incumbent workers, independently of whether or not they hire new workers. In order for such an effect to occur, three conditions would need to be satisfied. First, the firm must have initial wages below the prevailing wage. This is unlikely to be true for many firms in our sample, as we find that procurement auction participants (both winners and losers) have higher than average wages in the pre-period. Second, even if the winning firms had initial wages below the prevailing wage, prevailing wage laws would only bind if the new wage after winning the procurement contract would not have met the prevailing wage in the absence of a prevailing wage law. In the presence of an upward-sloping labor supply curve, the wage increase required to recruit new workers may reach the prevailing wage even among winners that did not initially pay the prevailing wage. Third, the procurement contract must be funded by a state government that currently has a prevailing wage law, or be funded by the federal government.

To investigate if there are actually effects of prevailing wage laws, we use repeals of state prevailing wage laws (United States Department of Labor, 2021) in a difference-in-differences analysis at the state-level to examine how wage and non-wage compensation are impacted by prevailing wages. For outcome measures, we use the EC data described above on wages and non-wage fringe benefits in the construction sector. In Table A.8, the difference-in-differences estimates for repeals suggest that prevailing wage laws have little to no effect on total compensation, wages, non-wage fringe benefits, or the share of total compensation from non-wage fringe benefits.[4] For example, the effect of a repeal on the log wage in the construction industry is 0.009 with standard error 0.029, and the effect on

[3] The list of states that currently have prevailing wage laws as well as the history of repeals is provided by the US Department of Labor here: https://www.dol.gov/agencies/whd/state/prevailing-wages.

[4] Prior work, reviewed by Duncan and Ormiston (2019), has found little evidence that prevailing wage laws increase wages in the construction industry. In their study of the first 9 repeals of prevailing wage laws and outcomes only through 1993, Kessler and Katz (2001) find economically small impacts on wages that become statistically insignificant when controlling for pre-trends. Our analysis includes more recent repeals, effects on non-wage outcomes, and uses modern difference-in-differences estimators for staggered treatment contexts (Callaway and Sant'Anna, 2020).

log non-wage fringe benefits in the construction industry is 0.015 with standard error 0.031.

3. *Safety Regulations and Procurement Auctions*

The construction industry is governed by extensive safety regulations. For example, construction employers in California must comply with Cal/OSHA regulations found in the following subchapters of California Code of Regulations, title 8, chapter 4: subchapter 4 (Construction Safety Orders); subchapter 5 (Electrical Safety Orders); and subchapter 7 (General Industry Safety Orders).[5] These regulations are typically task-specific, e.g., California requires certification and various safety procedures to be followed by crane operators.

It is important to observe that these safety regulations apply to all firms in the construction industry. In particular, government procurement projects are governed by the same safety regulations as private projects. Thus, receiving a procurement contract does not change the safety regulations governing the construction firm. Similarly, we have read the annual reports of public construction firms to examine the language they use to describe participation in government projects.[6] While they discuss the costs and opportunities associated with government contracts, we find no mention of changing safety policies in consideration of procurements.[7]

While we find no legal requirement that safety must be improved in response to winning a procurement auction, we may still worry that winning firms make safety improvements. In order to check for such effects, we download publicly-available data from OSHA safety inspections and link it to our procurement auction records. Since each state provides OSHA data in a different format, we focused on the largest state in our sample, California. Given our linked dataset between procurement auction and OSHA records, we then used these data to run the same regression specification as our baseline research design in equation (21), but now using safety investigations and violations as the outcome variables.

Reassuringly, we find fairly precisely estimated zero effects on the probability of a safety violation and on the probability of a safety investigation: As shown in Table A.7, the point estimates are 0.000 and 0.009, with standard errors 0.006 and 0.008 respectively, for the safety violation and investigation probabilities. These estimates suggest little if any impacts. By comparison, the probability of a safety violation is 4.1% and the probability of an investigation is 7.5% in the year that a firm is a bidder (both winners and losers) in an auction.

[5]See https://www.dir.ca.gov/dosh/construction-guide-summary.html.

[6]We accessed annual reports using https://www.annualreports.com. E.g., the annual report for Granite Construction Inc, one of the the largest auction winners in the procurement auction records for California, is available at https://www.annualreports.com/HostedData/AnnualReports/PDF/NYSE_GVA_2022.pdf.

[7]Although receiving a procurement contract does not lead to different safety regulations, a history of serious workplace safety violations may be a reason to deny a firm the chance to participate in a procurement auction at the pre-qualification phase.

4. Prevalence of Auctions with a Quality Dimension

Lewis and Bajari (2011) study a special type of auction, called an A+B auction, in which firms bid both a price and a time-to-completion. A bid submission contains a dollar amount, the "A" component, and a total number of days to complete the project, the "B" component. The score is a weighted sum of these two components, and the bidder with the lowest score wins. The A+B design provides an incentive for the bidder to disrupt traffic for a shorter period of time, and it is rarely used outside a few categories of construction projects that require road lane or shoulder closure.

The empirical context considered by Lewis and Bajari (2011) is California DOT procurement auctions during 2003-2008. They restrict the sample of auctions to the small set of project types that are more likely to use the A+B format. They also restrict the analysis to 5 districts (4, 6, 8, 11, and 12) that use the A+B format more frequently, with most auctions coming from the San Francisco Bay Area. Even with such selective sampling, among 708 auctions that fit the criteria, only 80 have the A+B format. The contracts auctioned through the A+B format are exceptionally large: the average engineer's estimate is $21.9 million for A+B auctions versus $4.6 million for other auctions.

To examine the prevalence of the A+B design in our analysis sample, we identify the list of A+B auctions in our 2001-2015 California DOT procurement auction data. In this sample, only 4.2% of auctions use the A+B format. We also check how these auctions enter our regressions. Recall that we only use auctions with a first-time winner to implement our estimation of the labor supply curve. In all of California during our sample period, only 12 auctions use the A+B format and are won by a first-time winner. Thus, A+B auctions comprise a very small share of our estimation sample for California.

Among the few auctions that use the A+B format in our sample, one may worry that the ultimate payment differs significantly from the total bid. According to Lewis and Bajari (2011), "standard contracts typically finish 7% early, whereas A+B contracts finish exactly on time." More specifically, in their sample, 52% of the A+B contracts are completed exactly on time. Completing on time means that the payment equals the part A bid. Using the A+B auctions that satisfy their sample time frame and districts, we find that about 87% of the total bids are determined by the A price component rather than the B time-to-completion component, suggesting that the price-only auction might remain a good approximation. We find similar statistics using the A+B auctions that fit our analysis sample criteria.

Another paper discussing auctions with a quality dimension is Takahashi (2018). He considers so-called design-bid auctions, in which each bidder submits a design and a price bid, and the price-per-quality score ratio determines the winner. In his sample from the Florida DOT between 2000 and 2011, only 152 auctions are design-bid auctions. In this case, quality-and-price-based auctions are again exceptions.

5. *Prevalence of Subcontracting*

Along with the project announcement, DOT publishes a detailed project description, including an engineer's estimate of the size and cost of each item. The winners of procurement auctions may subcontract some of those project items to be completed by other firms. However, data on subcontracting is limited, so there has been little empirical research on subcontracting.

The California DOT requires that bidders settle all subcontracting agreements prior to bid submission. Bidders must list all project items that will be subcontracted, along with the prices that will be paid to associated subcontractors. While subcontracting agreements are not available in a readily-usable data form, Balat, Komarova and Krasnokutskaya (2017) digitized this information from bidding documents submitted to the California DOT for projects auctioned between 2002 and 2016. Because their sample overlaps well with ours, their summary statistics represent ours well.

Using their data on subcontracting in California procurement auctions, Balat, Komarova and Krasnokutskaya (2017) find that subcontracting is common, with about 95% of auction winners subcontracting at least one item in road or bridge projects. However, the amount of subcontracting relative to the total value of the project is small. They find that, for the average (median) winner of a procurement contract, subcontractors only account for 8% (4%) of the bid value of the project. Thus, for the largest state in our data, subcontracting accounts for a relatively small share of the bid value.

F. Details on Labor Supply Elasticity Estimators

1. *Identification using the LMS Estimator*

Following Lamadon, Mogstad and Setzler (2022, LMS), we consider instrumenting for long-differences in log labor using short-differences in log value added (VA). Denoting the short-difference in log VA by $\Delta \text{va}_{jt} \equiv \log \text{VA}_{jt} - \log \text{VA}_{jt-1}$, the estimator of LMS is,

$$\widehat{\theta}_{\Delta \text{va}} \equiv \frac{\text{Cov}\left[w_{jt+e} - w_{jt-e'}, \Delta \text{va}_{jt}\right]}{\text{Cov}\left[\ell_{jt+e} - \ell_{jt-e'}, \Delta \text{va}_{jt}\right]}.$$

Unlike the estimators of Propositions 3–4, this estimator does not make use of information on procurement auctions and thus can be applied to the entire construction industry rather than only construction firms that bid for procurement projects.

Consider the following assumption, which compares short-run changes in VA to longer-run changes in TFP and firm-specific amenity shocks:

Assumption 1: *Suppose* $\exists e, e' > 0$ *sufficiently large such that (i)* $\phi_{jt+e} - \phi_{jt-e'}$ *is correlated with* Δva_{jt}*, and (ii)* Δva_{jt} *is orthogonal to* $\nu_{jt+e} - \nu_{jt-e'}$*.*

We then have the following result:

Proposition 6: *Under Assumption 1 and the rank condition* $Cov\left[\left(\ell_{jt+e}-\ell_{jt-e'}\right),\Delta va_{jt}\right]\neq 0$, $\widehat{\theta}_{\Delta va}$ *recovers* θ.

By equation (4),

$$\widehat{\theta}_{\Delta\text{va}}=\underbrace{\frac{\text{Cov}\left[\theta\left(\ell_{jt+e}-\ell_{jt-e'}\right),\Delta\text{va}_{jt}\right]}{\text{Cov}\left[\left(\ell_{jt+e}-\ell_{jt-e'}\right),\Delta\text{va}_{jt}\right]}}_{=\theta}+\underbrace{\frac{\text{Cov}\left[\left(\nu_{jt+e}-\nu_{jt-e'}\right),\Delta\text{va}_{jt}\right]}{\text{Cov}\left[\left(\ell_{jt+e}-\ell_{jt-e'}\right),\Delta\text{va}_{jt}\right]}}_{=0}=\theta,$$

where the denominator of each term is non-zero (i.e., the rank condition is satisfied) by Assumption 1(i) and the second term is zero by Assumption 1(ii). The key requirement, the exclusion condition $\text{Cov}\left[\left(\nu_{jt+e}-\nu_{jt-e'}\right),\Delta\text{va}_{jt}\right]=0$, relies on the assumption that firm-specific amenity shocks are transitory while TFP shocks are persistent. Thus, short-run VA growth is correlated with long-run employment growth (satisfying the rank condition due to persistence in TFP shocks), but orthogonal to long-run firm-specific amenity shocks. In practice, we must take a stand on the persistence of the transitory shocks. LMS argue that these transitory shocks are well-approximated as a moving average of order one, in which case, Assumption 1 holds as long as $e\geq 2, e'\geq 3$ and TFP shocks persist for at least e periods. We use the same choices of e and e' as LMS in our empirical implementation.

2. Implementation of RDD Estimators

We now describe the implementation of the RDD estimation defined in Proposition 4. Recall that, for a firm j that bids in auction ι at time t, we define the loss margin as $\tau_{jt}\equiv\frac{Z_{jt}-Z_\iota^*}{Z_\iota^*}$, where Z_ι^* is the winning bid in auction ι. We can interpret τ_{jt} as a measure of proximity to the discontinuity, satisfying $\tau_{jt}=0$ for auction winners ($D_{jt}=1$) and $\tau_{jt}>0$ for auction losers ($D_{jt}=0$). The regression specification is,

(A.47)

$$1\{\tau_{jt}\leq\overline{\tau}\}\,w_{jt+e}=\underbrace{\sum_{e'\neq\bar{e}}1\{\tau_{jt}\leq\overline{\tau}\}\,1\{e'=e\}\,\mu_{te'}^{\overline{\tau}}}_{\textit{event time fixed effect}}+\underbrace{\sum_{j'}\sum_{\iota'}1\{\tau_{jt}\leq\overline{\tau}\}\,1\{j'=j\text{ and }\iota'=\iota\}\,\psi_{j'\iota't}^{\overline{\tau}}}_{\textit{firm-auction fixed effect}}$$
$$+\underbrace{\sum_{e'\neq\bar{e}}1\{\tau_{jt}\leq\overline{\tau}\}\,1\{e'=e\}\,D_{jt}\lambda_{te'}^{\overline{\tau}}}_{\textit{treatment status by event time}}+\underbrace{1\{\tau_{jt}\leq\overline{\tau}\}\,\epsilon_{jte}}_{\textit{residual}},$$

where we have fully interacted the regression with the indicator $1\{\tau_{jt}\leq\overline{\tau}\}$ to remove any control units that do not place close bids to the winning firms. The parameter $\lambda_{te}^{\overline{\tau}}$ recovers the numerator of $\theta_{\text{RDD}}(\overline{\tau})$ for a particular choice of $\overline{\tau}$, pair (e,t), and $\bar{e}=-2$ is the omitted event time. As in the baseline implementation, we estimate $\lambda_{te}^{\overline{\tau}}$ for all t and e and then average across t, using the delta method to compute standard errors. The analogous regression in which ℓ_{jt+e} is the outcome recovers the denominator of $\theta_{\text{RDD}}(\overline{\tau})$. We average across event times e to form the main estimate.

As an alternative implementation of the RDD estimator defined in Proposition 4, consider a regression that controls for the loss margin, rather than restricting the sample based on the loss margin, as,

$$w_{jt+e} = \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} \mu_{te'}}_{\textit{event time fixed effect}} + \underbrace{\sum_{j'} \sum_{\iota'} 1\left\{j' = j \text{ and } \iota' = \iota\right\} \psi_{j'\iota' t}}_{\textit{firm-auction fixed effect}} + \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} D_{jt}\lambda_{te'}}_{\textit{treatment status by event time}} + \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} \varrho_e(\tau_{jt})}_{\textit{polynomial in loss margin}} + \underbrace{\epsilon_{jte}}_{\textit{residual}}, \tag{A.48}$$

where $\varrho_e(\tau_{jt})$ is an event time-specific polynomial in the loss margin τ_{jt}. In practice, we consider a linear specification, $\varrho_e(\tau_{jt}) = \varphi_e \tau_{jt}$, and a third-order polynomial specification, $\varrho_e(\tau_{jt}) = \varphi_{1e}\tau_{jt} + \varphi_{2e}\tau_{jt}^2 + \varphi_{3e}\tau_{jt}^3$.

3. *Implementation of Local Labor Market Estimators*

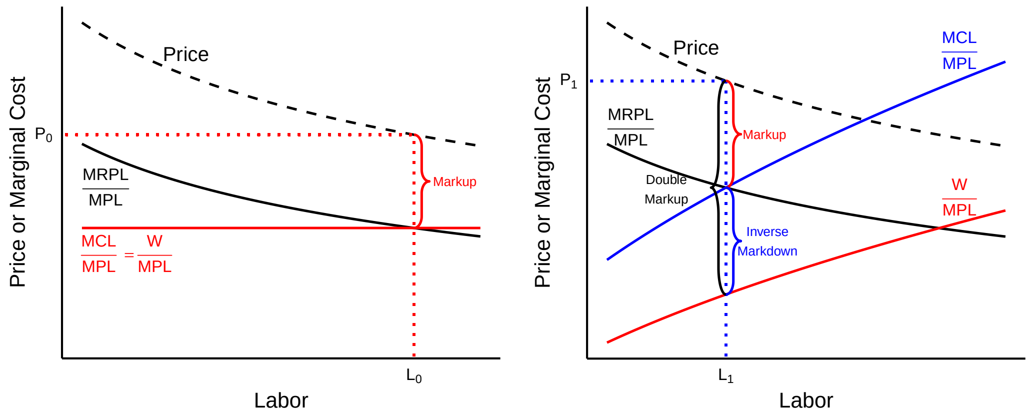
Let m denote the market in which the firm participates, and let $\mathcal{J}_m$ denote the set of firms that participate in market m. Possible markets include the auction in which the firm bids or the commuting zone in which the firm employs workers. Our goal is to estimate θ_{DiD} while controlling for market-specific shocks. To do so, we will implement the Proposition 3 estimator separately by market, allowing each market to experience its own sequence of event time effects.

Consider the cohort of firms that receive a procurement contract in year t ($D_{jt} = 1$) and the set of comparison firms that bid for a procurement in year t but lose ($D_{jt} = 0$). Let e denote an event time relative to t. For each event time $e = -4, ..., 4$, our DiD estimation for market m is implemented as

$$w_{jt+e} = \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} \mu_{te'}^{m}}_{\textit{market-specific event time fixed effect}} + \underbrace{\sum_{j'} 1\left\{j' = j\right\} \psi_{j't}}_{\textit{firm fixed effect}} + \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} D_{jt}\lambda_{te'}^{m}}_{\textit{treatment status by event time}} + \underbrace{\epsilon_{jte}}_{\textit{residual}} \quad \text{among} \quad j \in \mathcal{J}_m \tag{A.49}$$

where, for market m, λ_{te}^{m} recovers the numerator of θ_{DiD}^{m} for a particular pair (e, t) and $\bar{e} = -2$ is the omitted event time. This estimator differs from the baseline specification in that it only considers comparison firms that participate in the same market as the firm that receives a procurement contract, ensuring that market-specific shocks are controlled. In particular, market-specific shocks are captured by the μ_{te}^{m} parameters. The analogous regression in which ℓ_{jt+e} is the outcome recovers the denominator of $\theta_{\mathrm{DiD}}^{m}$. We average across event times e to form the main estimate. Finally, we average $\theta_{\mathrm{DiD}}^{m}$ across all of the markets m to form the overall θ_{DiD} estimate that controls flexibly for market-specific shocks.

G. Additional Tables and Figures



(a) Markup without Labor Market Power (b) Markup with Labor Market Power

Figure A.2. : Visualizing the Double Markup

Notes: This figure visualizes how prices are marked up due to the interaction between labor and product market power. It is constructed by simulating the solution to the firm's problem, omitting the procurement auctions. For simplicity, we parameterize the production function as $Q_{jt} = \Phi_{jt} L_{jt}$, although the results in the text hold for more general production functions. The curve labeled Price is the inverse product demand curve, and the productivity-adjusted MRPL curve is related to the price curve by $\frac{\text{MRPL}}{\text{MPL}} = (1-\epsilon) \times$ Price. The curve labeled $\frac{\text{W}}{\text{MPL}}$ is the productivity-adjusted wage, which is related to the productivity-adjusted MCL by $\frac{\text{MCL}}{\text{MPL}} = (1+\theta) \times \frac{\text{W}}{\text{MPL}}$. In subfigure (a), there is no labor market power ($\theta = 0$), so MCL and the wage are identical and the only markup is $(1-\epsilon)^{-1}$. In subfigure (b), there is labor market power ($\theta > 0$), generating a wedge between MCL and the wage whose size is determined by the inverse markdown, leading to an additional markup relative to the wage.

State	DOT Auction Records: Data Source	DOT Auction Records: Includes EIN	Final Sample: Linked Auction-Tax Data: Bidders in 2010	Share of 2010 Construction Sector: Value Added	Share of 2010 Construction Sector: FTE Workers
AL	State Website	✗	196	15.7%	17.4%
AR	State Website	✗	149	7.9%	12.8%
AZ	No	✗	*	*	*
CA	State Website	✗	1,041	8.3%	11.2%
CO	FOIA Request	✓	241	12.6%	14.7%
CT	FOIA Request	✗	126	9.4%	15.5%
FL	State Website	✓	344	30.7%	10.6%
GA	BidX Website	✗	137	4.3%	7.0%
IA	BidX Website	✗	256	15.4%	20.7%
ID	BidX Website	✗	112	17.2%	13.6%
IL	No	✗	*	*	*
IN	State Website	✓	213	10.6%	16.6%
KS	BidX Website	✓	130	13.7%	21.6%
KY	No	✗	*	*	*
LA	BidX Website	✗	167	11.5%	10.8%
MA	No	✗	*	*	*
MD	No	✗	*	*	*
ME	BidX Website	✗	141	13.7%	16.9%
MI	BidX Website	✗	391	9.5%	16.3%
MN	BidX Website	✗	262	13.5%	19.8%
MO	BidX Website	✗	179	14.9%	13.3%
MS	No	✗	*	*	*
MT	FOIA Request	✗	122	15.0%	23.6%
NC	BidX Website	✗	135	5.2%	9.8%
ND	FOIA Request	✗	*	*	*
NE	No	✗	*	*	*
NH	No	✗	*	*	*
NJ	No	✗	*	*	*
NM	BidX Website	✗	*	*	*
NV	No	✗	*	*	*
NY	No	✗	*	*	*
OH	BidX Website	✗	320	43.7%	17.5%
OK	No	✗	*	*	*
OR	No	✗	*	*	*
PA	No	✗	*	*	*
SC	No	✗	*	*	*
SD	No	✗	*	*	*
TN	BidX Website	✗	140	5.3%	11.5%
TX	FOIA Request	✓	551	4.9%	9.6%
UT	No	✗	*	*	*
VA	BidX Website	✗	241	14.2%	12.0%
VT	BidX Website	✗	*	*	*
WA	BidX Website	✗	200	7.5%	14.0%
WI	BidX Website	✗	194	12.1%	14.6%
WV	BidX, State Websites	✓	103	13.7%	19.0%
National			6,792	10.7%	9.9%

Table A.1—: Summary of Auction Data by State

Notes: The first two columns provide information on in-state DOT data sources by state, where "state" refers to the state in which the auction occurred. The first column indicates the source from which we obtained data on that state's DOT auctions, and the second column indicates whether or not EINs were included in the auction records. The final three columns provide information on the final sample of firms in the matched auction-tax data, where "state" refers to the state in which the firm filed taxes. Among firms in the construction industry in 2010, the last two columns consider the share of value added and FTE workers due to the firms that participated in auctions in our sample. We drop from these calculations firms that have missing values on the variables displayed, so the total sample size must be smaller than in Table A.2. An asterisk (*) denotes that number of bidders is non-zero but below the disclosure threshold.

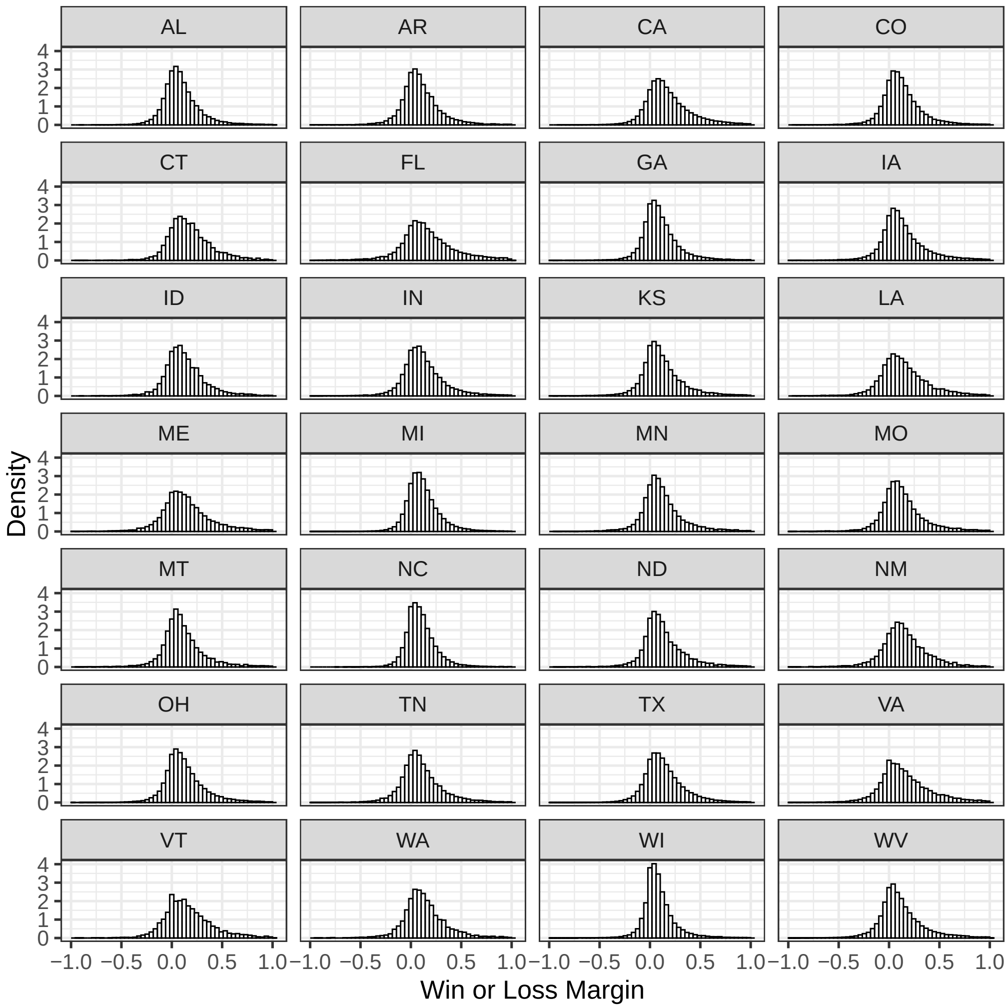
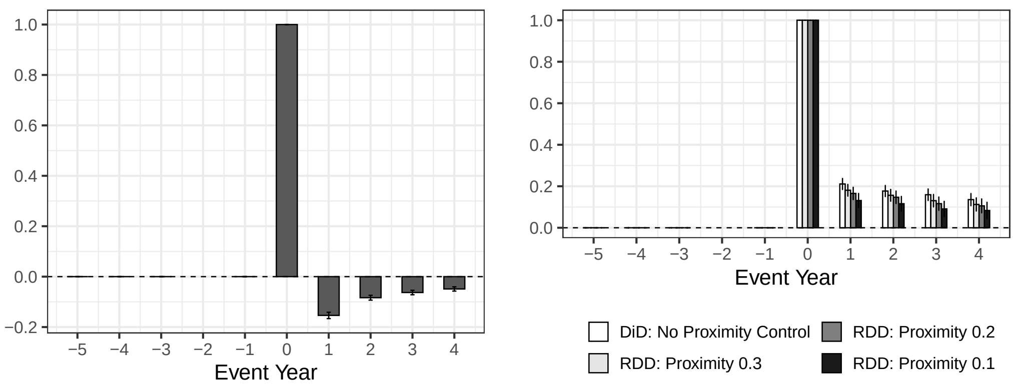


Figure A.3. : Chassang et al. (2022) Visual Test for Collusion

Notes: This figure displays the histogram of bid competition for each of the 28 states in our sample. Negative values indicate the difference between the winner's bid and the bid of the runner-up. Positive values indicate the difference between each loser's bid and the winner's bid. Differences are scaled by the winner's bid in each case. Chassang et al. (2022) demonstrate that, under some assumptions on the auction environment, these differences should display discontinuities in the histogram near zero if there is collusion.



(a) Indicator for First-time Winning (b) Indicator for Any Wins

Figure A.4. : Visualizing the Research Design

Notes: This figure presents estimates based on equation (21) of the impacts of winning a procurement contract. The omitted event time is −2. The outcomes considered are the probability of winning an auction for the first time (subfigure a) and the probability of winning any auction in the current year (subfigure b). It provides these estimates separately by event year. Proximity refers to the largest value of τ_{jt} permitted in the sample. 95% confidence intervals are displayed in brackets. Specification details and sample definitions are provided in the main text.

	Sample Size		Share of the Construction Industry
Number of Firms	7,876		0.9%
Workers per Firm	46		11.7%
	Value Per Firm ($ millions)	Mean of Log	Share of the Construction Industry
Sales	19.927	15.061	12.1%
EBITD	9.232	14.075	9.6%
Intermediates	14.661	14.719	12.4%
Wage bill	2.737	13.549	13.4%

Table A.2—: Sample Characteristics

Notes: This table displays descriptive statistics for the sample of firms that place bids in 2010. The third column compares aggregates for this sample to all firms in the construction industry in the 2010 tax records.

	Effect on Employment		Effect on Earnings		Implications for Labor Market		
	Estimate	Std. Err.	Estimate	Std. Err.	Parameter θ	Elasticity $1/\theta$	Markdown $(1+\theta)^{-1}$
Panel A. By Proximity:							
Any Proximity	0.083	(0.019)	0.020	(0.008)	0.245	4.084	0.803
Proximity 1.0	0.083	(0.019)	0.021	(0.008)	0.251	3.991	0.800
Proximity 0.5	0.080	(0.019)	0.020	(0.008)	0.251	3.980	0.799
Proximity 0.4	0.079	(0.020)	0.022	(0.008)	0.277	3.608	0.783
Proximity 0.3	0.079	(0.020)	0.022	(0.008)	0.281	3.559	0.781
Proximity 0.2	0.079	(0.021)	0.020	(0.009)	0.257	3.892	0.796
Proximity 0.1	0.065	(0.025)	0.019	(0.010)	0.286	3.491	0.777
Panel B. By Proximity for Stayers:							
Any Proximity	0.083	(0.019)	0.023	(0.006)	0.278	3.600	0.783
Proximity 1.0	0.083	(0.019)	0.024	(0.006)	0.283	3.530	0.779
Proximity 0.5	0.080	(0.019)	0.022	(0.006)	0.277	3.605	0.783
Proximity 0.4	0.079	(0.020)	0.023	(0.006)	0.294	3.403	0.773
Proximity 0.3	0.079	(0.020)	0.023	(0.006)	0.288	3.467	0.776
Proximity 0.2	0.079	(0.021)	0.021	(0.007)	0.271	3.689	0.787
Proximity 0.1	0.065	(0.025)	0.019	(0.007)	0.286	3.499	0.778
Panel C. By Worker Incumbency:							
Stayer Spell: $(-1, ..., 1)$	0.083	(0.019)	0.023	(0.005)	0.272	3.681	0.786
Stayer Spell: $(-2, ..., 2)$	0.083	(0.019)	0.023	(0.006)	0.278	3.600	0.783
Stayer Spell: $(-3, ..., 3)$	0.083	(0.019)	0.021	(0.007)	0.253	3.957	0.798
Tenure: 1 Year	0.083	(0.019)	0.023	(0.006)	0.277	3.615	0.783
Tenure: 2 Years	0.083	(0.019)	0.023	(0.006)	0.277	3.615	0.783
Tenure: 3 Years	0.083	(0.019)	0.025	(0.006)	0.301	3.326	0.769
Tenure: 4 Years	0.083	(0.019)	0.022	(0.006)	0.266	3.766	0.790
Panel D. By Employment Intensity:							
Add Indep. Contractors	0.092	(0.021)	0.020	(0.009)	0.220	4.548	0.820
110% of FTE Wage	0.083	(0.019)	0.023	(0.006)	0.276	3.627	0.784
120% of FTE Wage	0.083	(0.019)	0.022	(0.006)	0.266	3.755	0.790
130% of FTE Wage	0.083	(0.019)	0.022	(0.006)	0.264	3.788	0.791
140% of FTE Wage	0.083	(0.019)	0.021	(0.006)	0.256	3.909	0.796
150% of FTE Wage	0.083	(0.019)	0.019	(0.006)	0.230	4.346	0.813
Panel E. Interacted DiD Designs:							
Fully-interacted: CZ	0.074	(0.019)	0.017	(0.009)	0.224	4.467	0.817
Fully-interacted: Auction	0.088	(0.027)	0.018	(0.009)	0.206	4.846	0.829
Panel F. LMS Design for All Construction Firms:							
Baseline	0.087		0.023		0.266	3.758	0.790
Control CZ	0.092		0.027		0.299	3.349	0.770

Table A.3—: Specifications for Estimating the Parameter θ

Notes: This table presents estimates of the impacts of winning a procurement contract on employment and earnings per worker in the post-treatment time period. Employment and earnings per worker are measured in log units. Proximity refers to the largest value of τ_{jt} permitted in the sample. Specification details and sample definitions are provided in the main text. Standard errors are not available for the estimates in Panel F.

Design:	DiD	RDD		
Proximity:	Any	0.3	0.2	0.1
Impact: Before Treatment	-0.003	0.003	0.003	-0.007
	(0.013)	(0.019)	(0.020)	(0.024)
Impact: After Treatment	0.010	0.017	0.016	0.009
	(0.011)	(0.017)	(0.018)	(0.021)

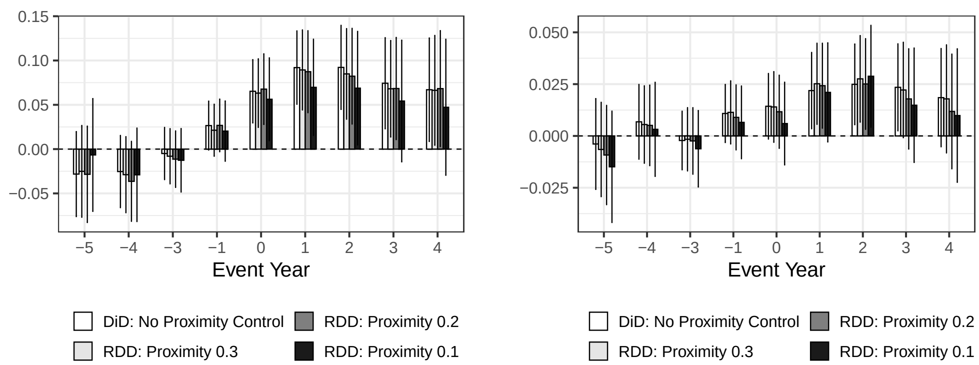
Table A.4—: Impacts of Winning a Procurement Contract: Earnings of New Hires

Notes: This table presents estimates of the impacts of winning a procurement contract on the log earnings per worker at the new employer among new hires in the post-treatment and pre-treatment time periods. Proximity refers to the largest value of τ_{jt} permitted in the sample. Specification details and sample definitions are provided in the main text.

Design:	DiD	RDD		
Proximity:	Any	0.3	0.2	0.1
Impact: Before Treatment	-0.002	-0.009	-0.003	0.004
	(0.019)	(0.027)	(0.029)	(0.034)
Impact: After Treatment	0.009	0.012	0.014	0.023
	(0.016)	(0.024)	(0.026)	(0.030)

Table A.5—: Impacts of Winning a Procurement Contract: Quality of New Hires

Notes: This table presents estimates of the impacts of winning a procurement contract on worker quality. Worker quality is measured by log earnings per worker in the previous firm. Proximity refers to the largest value of τ_{jt} permitted in the sample. Specification details and sample definitions are provided in the main text.

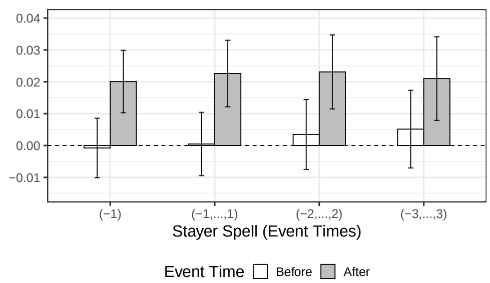


(a) Outcome: Log Number of Employees

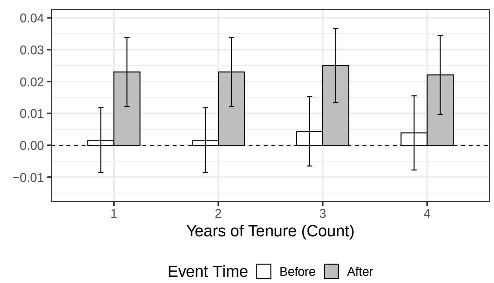
(b) Outcome: Log Earnings per Employee

Figure A.5. : DiD and RDD Estimates of the Effects of Winning a Procurement

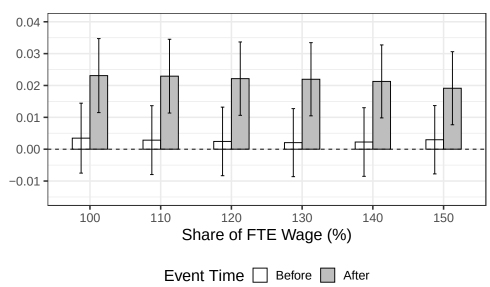
Notes: This figure displays estimates of the effects of winning a procurement contract on winners relative to losers across event times. The winner is announced during event time 0, and outcomes are normalized to zero for both winners and losers in event time -2. All specifications control for time-invariant auction and firm characteristics as well as time fixed effects. The outcomes are log employment in subfigure (a) and log earnings per employee in subfigure (b). Proximity refers to the largest value of τ_{jt} permitted in the sample. 95% confidence intervals are displayed in brackets.



(a) Stayer Definitions



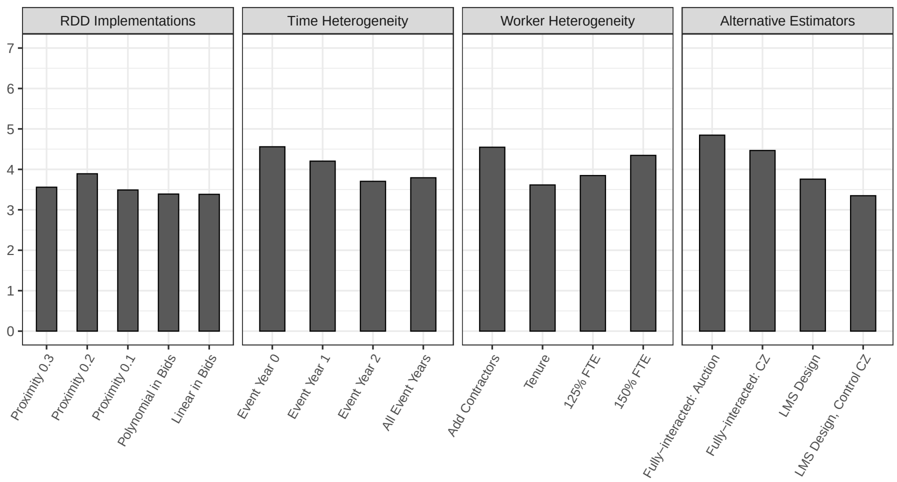
(b) Tenure Definitions



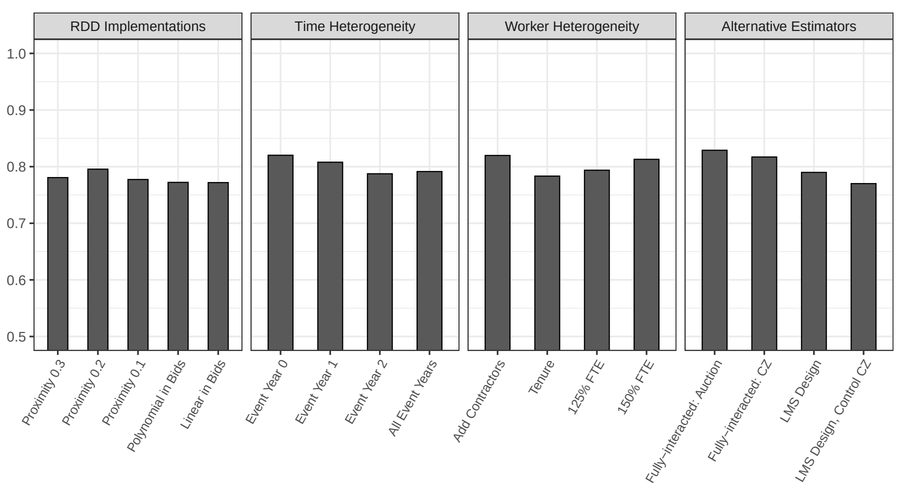
(c) FTE Definitions

Figure A.6. : Impacts on Log Earnings per Worker: Sensitivity to Worker Sample

Notes: This figure presents estimates based on equation (21) of the impacts of winning a procurement contract. The outcome considered is log earnings per worker. It provides these estimates for alternative sample definitions for stayers (subfigure (a)), tenured workers (subfigure (b)), and full-time equivalence (FTE) thresholds as a percentage of the annualized minimum wage (subfigure (c)). 95% confidence intervals are displayed in brackets. Specification details and sample definitions are provided in the main text.



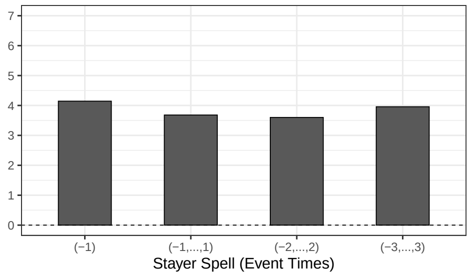
(a) Labor Supply Elasticity, $1/\theta$



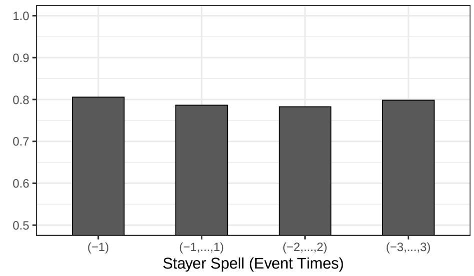
(b) Wage Markdown relative to MRPL, $(1+\theta)^{-1}$

Figure A.7. : Specification Checks: Estimates of the Labor Supply Elasticity and Wage Markdown

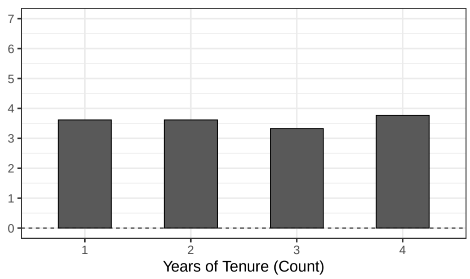
Notes: This figure presents the sensitivity checks for the estimates of the labor supply elasticity, $1/\theta$, and the wage markdown relative to MRPL, $(1+\theta)^{-1}$. Specification details and sample definitions are provided in the main text.



(a) Stayer Definitions: Labor Supply Elasticity



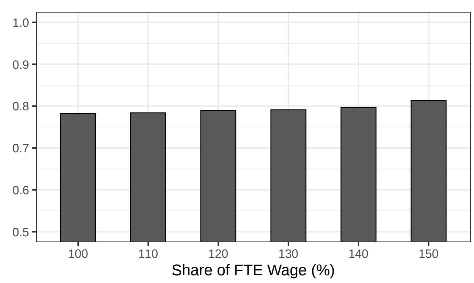
(b) Stayer Definitions: Wage Markdown



(c) Tenure Definitions: Labor Supply Elasticity

(d) Tenure Definitions: Wage Markdown

(e) FTE Definitions: Labor Supply Elasticity



(f) FTE Definitions: Wage Markdown

Figure A.8. : Labor Supply Elasticity and Wage Markdown: Sensitivity to Worker Sample

Notes: This figure presents estimates of the labor supply elasticity, $1/\theta$, and the wage markdown relative to MRPL, $(1+\theta)^{-1}$ for alternative sample definitions for stayers (subfigures (a)-(b)), tenured workers (subfigures (c)-(d)), and full-time equivalence (FTE) thresholds as a percentage of the annualized minimum wage (subfigures (e)-(f)). Specification details and sample definitions are provided in the main text.

States:	All		Prevailing Wage	
Workers:	All	Stayers	All	Stayers
Impacts of Winning an Auction:				
Log Employment:	0.083		0.081	
	(0.019)		(0.023)	
Log Earnings per Worker:	0.020	0.023	0.023	0.027
	(0.008)	(0.006)	(0.010)	(0.007)
Implied Labor Parameters:				
Labor Supply Elasticity:	4.084	3.600	3.508	3.054
Markdown relative to MRPL:	0.803	0.783	0.778	0.753

Table A.6—: Impacts of Winning a Procurement Contract: Prevailing Wage States

Notes: This table presents estimates of the impacts of winning a procurement contract on log employment and log earnings per worker in the post-treatment time period. It provides the impacts on log earnings per worker separately for all workers in the firm and stayers. It compares all states to states that have a state-specific prevailing wage requirement. Specification details and sample definitions are provided in the main text.

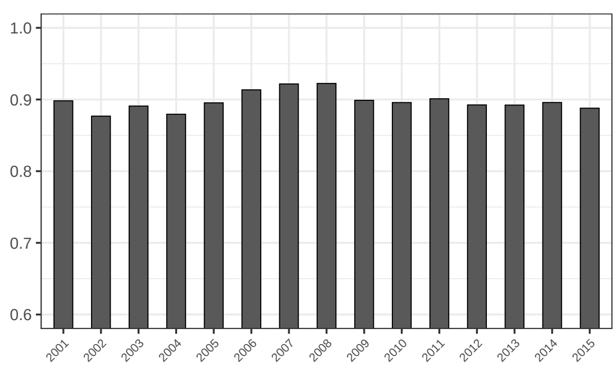


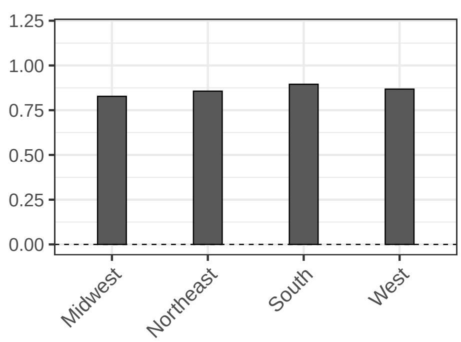
Figure A.9. : Interquartile Range in TFP by Year

Notes: This figure presents the interquartile range of TFP estimated separately by calendar year. Specification details and sample definitions are provided in the main text.

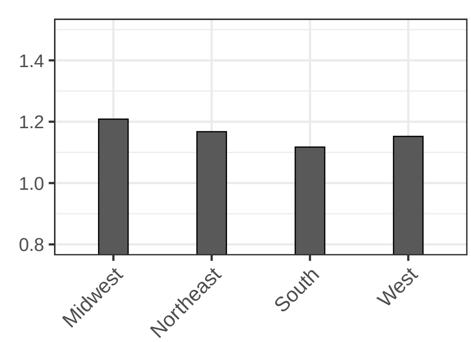
	OSHA Investigations		OSHA Violations	
	Probability	Count	Probability	Count
	Occurrence			
Observed Average:	0.075	0.139	0.041	0.110
	Impacts of Winning a Procurement Auction			
Impact: Before Treatment	0.000	-0.012	0.000	-0.009
	(0.006)	(0.016)	(0.004)	(0.018)
Impact: After Treatment	0.009	0.004	0.000	-0.006
	(0.008)	(0.020)	(0.006)	(0.023)

Table A.7—: Impacts of Winning a Procurement Contract: OSHA Safety Outcomes in California

Notes: This table presents estimates of the impacts of winning a procurement contract on OSHA safety investigations and violations in California during 2001-2015. The observed average is reported for bidders (winners and losers) in the years of active bidding.



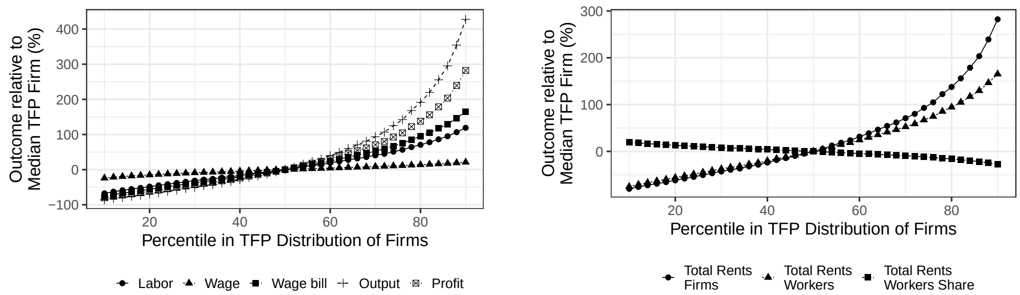
(a) Estimate of $1-\epsilon$



(b) Implied Price Markup

Figure A.10. : Heterogeneity across Census Regions in the Estimate of $1-\epsilon$

Notes: This figure presents heterogeneity across Census regions in the estimate of $1-\epsilon$ using the estimator in equation (25), as well as the implied price markup $(1-\epsilon)^{-1}$.



(a) Outcomes (b) Total Rents

Figure A.11. : Outcomes and Rents for Alternative TFP Percentiles, among Firms without Procurement Contracts ($D_{jt} = 0$)

Notes: In this figure, we assign alternative TFP quantiles to the median-TFP firm in the $D_{jt} = 0$ sample without changing any other primitives of the model, then re-solve the model to obtain this firm's alternative outcomes and rents. The x-axis displays the alternative TFP assigned to the firm (as a percentile in the population TFP distribution). Each y-axis value is expressed as a percent change relative to the actual value for the median-TFP firm.

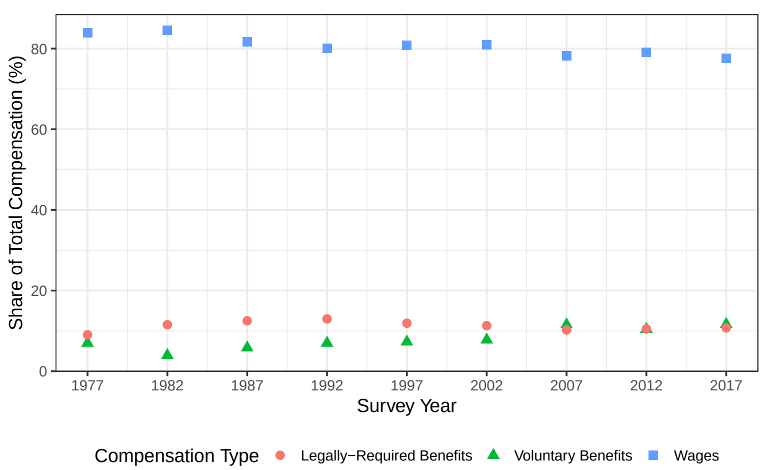


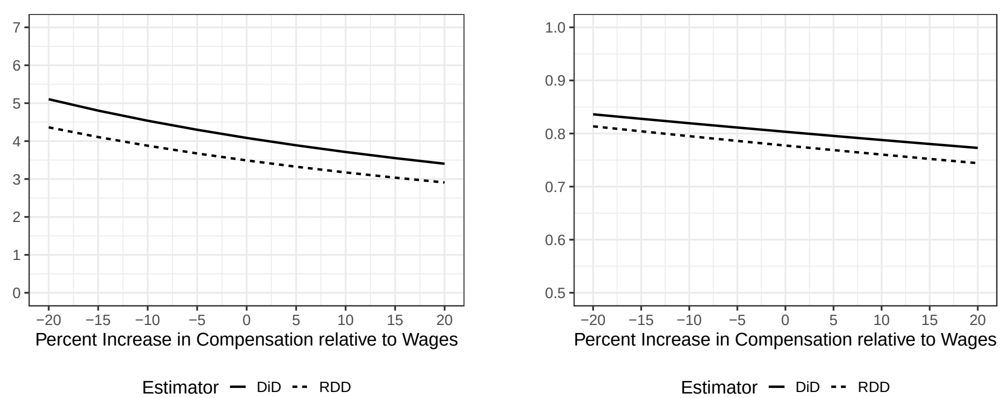
Figure A.12. : Survey Evidence from the Economic Census of the Construction Sector: Sources of Compensation in the Construction Industry

Notes: This figure uses survey data from the Economic Census of the Construction Sector to estimate the share of total compensation from legally-required benefits, voluntary benefits, and wages. See Appendix E.1 for details.

Total Compensation (log)	Wage Compensation (log)	Non-wage Fringe Benefits (log)	Share Non-wage Fringe Benefits (fraction)
Difference-in-Differences for State Davis-Bacon Repeals			
0.009	0.009	0.015	0.000
(0.026)	(0.029)	(0.031)	(0.005)

Table A.8—: Impacts of Repeals of State Prevailing Wage Laws

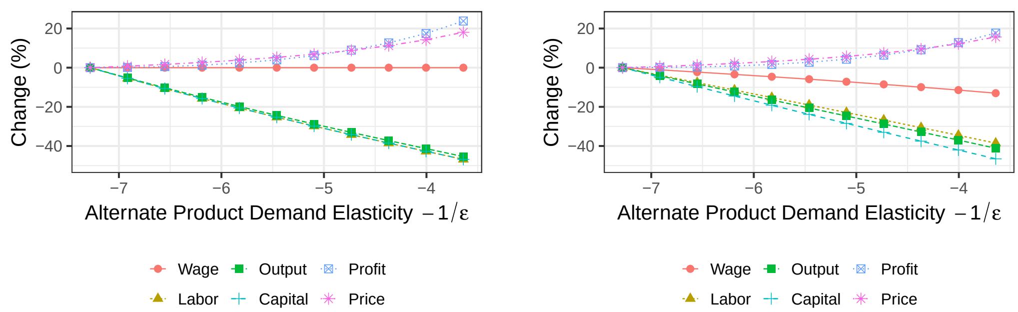
Notes: This table combines information on repeals of state prevailing wage laws with survey data from the Economic Census of the Construction Sector to estimate the impacts of prevailing wage law repeals on wage and non-wage compensation. See Appendix E.2 for details.



(a) Labor Supply Elasticity (b) Wage Markdown relative to MRPL

Figure A.13. : Sensitivity to Allowing Endogenous Amenity Responses to Winning a Procurement Auction

Notes: In this figure, we allow for amenity responses to winning a procurement auction and adjust the θ_{DiD} and θ_{RDD} estimates to account for these amenity responses. See Appendix H for details.



(a) Impacts of Product Market Power without Labor Market Power ($\theta = 0$)

(b) Impacts of Product Market Power with Labor Market Power (True θ)

Figure A.14. : Estimated Impacts of Changes in Product Market Power

Notes: This figure presents expected impacts of increased product market power, using the approach defined in Section IIB. To do so, it simulates from the model defined in Section I for the typical firm, evaluated at the parameter estimates provided in Table 2. In subfigure (a), we assume there is only market power in the product market, while in subfigure (b), we assume market power in both labor and product markets is characterized by our preferred estimates. Each simulated outcome is expressed as a percent change relative to the value observed in the data.

H. Sensitivity to Assuming there are Causal Effects of Winning an Auction on Amenities

Let Comp_{jt} denote the total compensation offered by the firm (inclusive of wages and amenities). If we observed Comp_{jt}, we could infer the (inverse) labor supply elasticity with respect compensation from the estimand

$$\widetilde{\theta} = \frac{\mathbb{E}\left[\Delta \log \text{Comp}_{jt} | \tau_{jt} = 0\right] - \mathbb{E}\left[\Delta \log \text{Comp}_{jt} | 0 < \tau_{jt} \leq \overline{\tau}\right]}{\mathbb{E}\left[\Delta \ell_{jt} | \tau_{jt} = 0\right] - \mathbb{E}\left[\Delta \ell_{jt} | 0 < \tau_{jt} \leq \overline{\tau}\right]}.$$

In practice, however, we only observe wages W_{jt}, so we use $\Delta \log W_{jt}$ in place of $\Delta \log \text{Comp}_{jt}$.

We now examine how the key conclusions regarding the labor supply curve would change if log compensation in reality increased more (or less) than log earning. To do so, it is useful to define

$$\lambda \equiv \frac{\mathbb{E}\left[\Delta \log \text{Comp}_{jt} | \tau_{jt} = 0\right] - \mathbb{E}\left[\Delta \log \text{Comp}_{jt} | 0 < \tau_{jt} \leq \overline{\tau}\right]}{\mathbb{E}\left[\Delta \log W_{jt} | \tau_{jt} = 0\right] - \mathbb{E}\left[\Delta \log W_{jt} | 0 < \tau_{jt} \leq \overline{\tau}\right]} - 1,$$

so that $\lambda \times 100\%$ is the percent increase in log compensation relative to log wages. If $\lambda = 0$, then the change in compensation is proportional to the change in wages. In this case, there is no bias in using wages in place of total compensation when estimating θ. However, if $\lambda > 0$ (or $\lambda < 0$), then the change in log compensation is greater (or smaller) than the change in log wages, so the estimate of θ using wages may be downward-biased (or upward-biased).

In Figure A.13, we calibrate $\lambda \times 100\%$ and examine how our conclusions would change if winning a procurement auction had a causal effect on amenity provision. We find that, even in the rather extreme case in which log compensation increases by 20% more (or less) than log earnings due to amenity responses, the labor supply curve is upward sloping with an elasticity that would be close to our baseline estimates in Figure 3. Furthermore, there is still a significant wage markdown relative to MRPL, and it is close to our baseline estimates using log wages.

I. Heterogeneity in TFP

We now investigate how the outcomes would change if the median-TFP firm instead had above-median or below-median TFP. To do so, we assign alternative TFP quantiles to this firm without changing any other primitives of the model, then re-solve the model to obtain this firm's alternative outcomes and rents.[8] The results are provided in Figure A.15. The x-axis displays the alternative draw of TFP assigned to the firm (as a percentile in the population TFP distribution).

[8]To solve the model for firms that currently receive procurement contracts, we must account for the optimal markups in the bids, requiring that we integrate across the distribution of opportunity costs (equation 14). To overcome the computational challenge, we implement the quantile representation method proposed by Luo (2020); implementation details are provided in Supplemental Appendix J.

In Figure A.15(a), the y-axis presents the firm's labor, wage, wage bill, output, and profits, and in Figure A.15(b), the y-axis presents the total rents of the firm, total rents of its workers, and workers' share of total rents. Each y-axis value is expressed as a percent change relative to the actual value for the median-TFP firm (reported in the first column of Table 4).

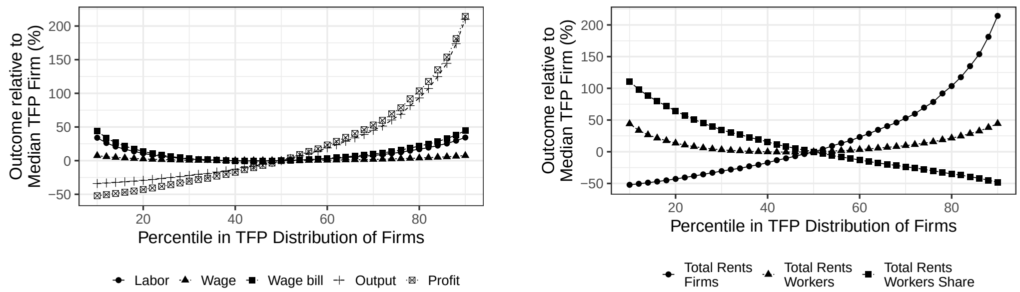
We find that, when the firm is more productive (above-median TFP), it chooses to produce more output, which requires hiring more workers. Since the labor supply curve is upward-sloping, it must bid up wages to increase employment, which also increases the wage bill. If TFP is set to the 75th percentile, the firm employs 12% more labor, pays 3% higher wages, and spends 15% more on labor. It produces 65% more output and earns 74% more profits. By contrast, if TFP is set to the 25th percentile, it produces 26% less output and earns 37% lower profit. If TFP is set below the 25th percentile, the firm also hires more workers and pays greater wages than with median TFP. This is because it needs to produce the minimum output specified by the government in the procurement contract, $\overline{Q}^G$, and must compensate for low productivity by hiring more labor than it would with median TFP.[9] Since firm rents increase more than worker rents as TFP increases, the share of rents captured by workers is decreasing in TFP. This result complements the recent literature on product market competition which has found that more productive firms have higher markups and lower labor shares (Autor et al., 2020; de Loecker, Eeckhout and Unger, 2020). We account for both labor and product market power in a constant-elasticity framework and find a lower rent share to workers at more productive firms.

J. Computational Details

Overview:

Simulating model outcomes is computationally challenging. Since $1/\theta$ and $-1/\epsilon$ both appear in the firm's opportunity cost $\sigma(\phi_{jt}, u_{jt})$ (recall the definition associated with equation 8), it follows that changing these parameters also changes the optimal bid Z^*_{jt} (equation 14). In turn, the bid affects the additional rents captured by firms from winning a procurement contract. To simulate from the model, we first solve the second stage problem for each ϕ_{jt} to find the distribution of opportunity costs. Next, we solve the first stage problem to obtain the distribution of optimal bids given the opportunity costs. Finally, we combine the optimal bid distribution from the first stage with the optimal private market profits from the second stage. From this, we recover all outcomes. To ease the computational burden in solving for these distributions we implement the quantile representation method of Luo (2020). Our main results focus on outcomes

[9]Figure A.11 provides a similar analysis but for the majority of firms that do not currently have a procurement contract and thus have no incidence of procurements. We find that wages, employment, the wage bill, and rents are monotonically increasing in TFP when shutting down the government market, confirming that the non-monotonicity in these outcomes in Figure A.15 is due to the constraint that $Q_{1jt} \geq \overline{Q}^G$.



(a) Outcomes of Workers and Firms (b) Total Rents of Workers and Firms

Figure A.15. : Outcomes and Rents for Alternative TFP Percentiles

Notes: In this figure, we assign alternative TFP quantiles to the median-TFP firm in the $D_{jt} = 1$ sample without changing any other primitives of the model, then re-solve the model to obtain this firm's alternative outcomes and rents. The x-axis displays the alternative TFP assigned to the firm (as a percentile in the population TFP distribution). Each y-axis value is expressed as a percent change relative to the actual value for the median-TFP firm (reported in the first column of Table 4).

for the typical firm (the firm with the median value of ϕ_{jt}), which further reduces the computational burden.

Second stage:

Denote the TFP quantile function as $\phi(\alpha)$ where, for example, $\alpha = 0.10$ indicates the 10th quantile of the TFP distribution. We use a log Normal distribution to approximate the distribution of TFP, which allows for a simple mapping between ϕ and α, choosing the standard deviation that matches the interquartile range of TFP (reported in Table 2). For each combination of winner status, TFP quantile, and auction size $\left(d, \alpha, \bar{Q}^G\right)$, we solve the second-stage problem for firm and worker outcomes. This is done by numerical optimization of the profit function (equation 8) subject to the labor supply curve (equation 2), the production function (equation 9), and optimal intermediate inputs (equation 10).

First stage:

The challenge is to compute expectations of the second-stage across the distribution of outcomes from the first-stage. To solve the first-stage, note that the opportunity cost of winning an auction of size $\overline{Q}^G$ is $\sigma\left(\alpha|\overline{Q}^G\right) = \pi_0^H(\alpha) - \pi_1^H\left(\alpha|\overline{Q}^G\right)$. Since π_{1jt}^H is the winning firm's revenue in the private market net of the total cost, it follows that $\pi_{0jt}^H > \pi_{1jt}^H$ and thus $\sigma > 0$. π_1^H is decreasing in $\overline{Q}^G$, and π_0^H does not depend on $\overline{Q}^G$. Moreover, σ is decreasing in α. In other words, a higher TFP firm has a lower opportunity cost of producing in the government procurement market. Since α represents quantiles of TFP, it has the standard uniform distribution. The probability that the winning quantile is less than α is the probability that it is the lowest among all I bidders' draws from the standard uniform distribution, yielding the probability α^I and associated density function $f_1(\alpha, I) = I\alpha^{I-1}$. By similar reasoning, the density function of a losing firm's TFP quantile is $f_0(\alpha, I) = \frac{I}{I-1}(1 - \alpha^{I-1})$.

Solution:

Let $Y_d\left(\alpha|\overline{Q}^G\right)$ denote a second-stage outcome for a firm characterized by TFP quantile α bidding in an auction of size $\overline{Q}^G$. Using the distribution functions from the first stage, we compute the expected outcome as $\mathbb{E}\left[Y_d|\overline{Q}^G, I\right] = \int_0^1 Y_d\left(\alpha|\overline{Q}^G\right) f_d(\alpha, I)\, d\alpha$. For example, the probability that a bidder with TFP ϕ_{jt} wins the project is the probability that its TFP is the highest among all participating bidders, i.e, $H(\phi_{jt})^I$,where H denotes the distribution of TFP. This implies that the density function of the winner's TFP is $IH(\phi_{jt})^{I-1}h(\phi_{jt})$. The profit function depends on who wins the auction, in particular, the TFP of the winner. The expected profit of the winner is then

$$\bar{\pi}_{1jt} = \int \pi_{1jt}(\phi_{jt}|\overline{Q}^G)[IH(\phi_{jt})^{I-1}h(\phi_{jt})]d\phi_{jt} = \int \pi_{1jt}(\phi_{jt}(\alpha)|\overline{Q}^G)I\alpha^{I-1}d\alpha.$$

Note that this expectation depends on the combinations $\left(\overline{Q}^G, I\right)$. One possibility is to solve the model for each possible combination of $\left(\overline{Q}^G, I\right)$, and then average across them. In our setting, this is computationally infeasible. An alternative is to evaluate $\left(\overline{Q}^G, I\right)$ at representative values. In practice, we choose the values of $\left(\overline{Q}^G, I\right)$ that provide the best fit to the additional rents from procurement projects, $(V_{jt\Delta}, \pi_{jt\Delta})$, for the typical firm. The best fit yields a model-simulated incidence on workers of about \$6,500, which is the same as the main estimate in Table 4, and incidence on firms of \$9,200, which is very close to the main estimate of about \$9,600 in Table 4. The implied incidence share on workers is about 41%, which is about the same as our main estimate. The best fit is achieved at $I = 5$ bidders per auction, which is in the right ballpark to the mean observed value in the data of around 8 bidders per auction.

Additional details:

We now provide the derivation of the quantile representation of the optimal bidding strategy. Consider a standard first-price auction model. Following Guerre et al. (2000), we can rewrite the first-order condition and obtain a representation of the cost as a function of observables:

$$c = b - \frac{1}{I-1}\frac{1 - \mathfrak{H}(b)}{\mathfrak{h}(b)},$$

where $\mathfrak{H}(\cdot)$ and $\mathfrak{h}(\cdot)$ are the bid distribution and density, respectively. Since the bidding strategy is strictly increasing, we can further rewrite this expression in terms of quantiles:

$$c(\alpha) = b(\alpha) - \frac{1}{I-1}[1-\alpha]b'(\alpha),$$

where $c(\cdot)$ and $b(\cdot)$ are the cost quantile function and the bid quantile function, respectively. The boundary condition is that the least efficient firm bids the highest, i.e., $c(1) = b(1)$. Following Luo (2020), we solve this ODE and obtain the mapping from the cost quantile function to the bid quantile function:

$$b(\alpha) = (I-1)(1-\alpha)^{1-I} \int_{\alpha}^{1} c(\tilde{\alpha})(1-\tilde{\alpha})^{I-2} d\tilde{\alpha}.$$

This representation is convenient for numerically solving the first-price procurement auction model.

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