Online Appendix: "Imperfect Competition, Compensating Differentials and Rent Sharing in the U.S. Labor Market"

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A. Details on Model Solutions

1. Derivation of equilibrium wages

Given the nested logit preferences and a given set of wages $\mathbf{W}_t = \{W_{jt}(X)\}_{j=1..J}$ we get that

$$Pr[j(i,t){=}j|X_i{=}X,\mathbf{W}_t] = \frac{\left(\sum_{j'\in J_{r(j)}}\left(\tau G_{j'}(X)\right)^{\beta/\rho_{r(j)}} W_{j't}(X)^{\lambda\beta/\rho_{r(j)}}\right)^{\rho_{r(j)}}}{\sum_{r'}\left(\sum_{j'\in J_{r'}}\left(\tau G_{j'}(X)\right)^{\beta/\rho_{r'}} W_{j't}(X)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \times \frac{\left(\tau G_{j}(X)\right)^{\beta/\rho_{r(j)}} W_{jt}(X)^{\lambda\beta/\rho_{r(j)}}}{\sum_{j'\in J_{r(j)}}\left(\tau G_{j'}(X)\right)^{\beta/\rho_{r(j)}} W_{j't}(X)^{\lambda\beta/\rho_{r(j)}}}$$

and

$$\mathbb{E}\left[u_{it}|X_i = X,\mathbf{W}_t\right] = \frac{1}{\beta}\left[\log\left(\sum_r\left(\sum_{j\in J_r}\left(\tau G_j(X)\right)^{\beta/\rho_r}\left(W_{jt}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r} + \overline{C}\right)\right],$$

where $\overline{C}$ is an unrecoverable constant. It is useful to introduce the following definition before stating the Lemmas:

$$C_r \equiv \frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}.$$

LEMMA 1: *Assume that firms believe they are strategically small. That is, in the firm's first order condition, we impose that*

$$\frac{\partial I_{rt}(X)}{\partial W_{jt}(X)} = 0.$$

We can then show that for firm j in market r

$$Y_{jt} = \left(A_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\left(H_{jt}\right)^{1-\alpha_r} \tag{1}$$

$$W_{jt}(X) = C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \tag{2}$$

$$L_{jt} = H_{jt} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}, \tag{3}$$

where H_{jt} is implicitly defined by

$$H_{jt} \equiv \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r}\, dX\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

and we define

$$K_{rt}(X) \equiv NM(X)\frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r},$$

$$I_{rt}(X) \equiv \left(\sum_{j'\in J_r}\left(\tau^{1/\lambda}G_{j'}(X)^{1/\lambda}C_r X^{\theta_{j'}}A_{j't}\left(\frac{Y_{j't}}{A_{j't}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)}.$$

Proof:We start from the firm's problem specified in the main text including the tax parameters. Using shorthand r for $r(j)$, we have

$$\max_{\{W_{jt}(X),D_{jt}(X)\}} A_{jt}\left(\int X^{\theta_j}D_{jt}(X)\mathrm{d}X\right)^{1-\alpha_r} - \int W_{jt}(X)D_{jt}(X)\mathrm{d}X$$

$$\text{s.t. } D_{jt}(X) = NM(X)\frac{I_{rt}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(G_j(X)^{1/\lambda}\tau^{1/\lambda}\frac{W_{jt}(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}$$

and define:

$$K_{rt}(X) \equiv NM(X)\frac{I_{rt}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}.$$

We substitute in the labor supply function and derive the first order condition with respect to $W_{jt}(X)$:

$$(1-\alpha_r)X^{\theta_j}\left(\frac{\lambda\beta}{\rho_r}W_{jt}(X)^{\lambda\beta/\rho_r-1} + \frac{1}{K_{rt}(X)}\frac{\partial K_{rt}(X)}{\partial W_{jt}(X)}W_{jt}(X)^{\lambda\beta/\rho_r}\right)\tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}$$
$$= \tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}\left(\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X)^{\lambda\beta/\rho_r} + \frac{1}{K_{rt}(X)}\frac{\partial K_{rt}(X)}{\partial W_{jt}(X)}W_{jt}(X)^{1+\lambda\beta/\rho_r}\right).$$

Under the assumption that $\frac{\partial I_{rt}(X)}{\partial W_{jt}(X)} = 0$, the first order condition simplifies to

$$\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X) = \frac{\lambda\beta}{\rho_r}(1-\alpha_r)X^{\theta_j}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}},$$

or

$$W_{jt}(X) = C_r X^{\theta_j}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}.$$

Turning to the output of the firm,

$$
\begin{aligned}
Y_{jt}/A_{jt} &= \left(\int X^{\theta_j} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} W_{jt}(X)^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int \left(X^{\theta_j}\right)^{1+\lambda\beta/\rho_r} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} \left(C_r A_{jt}\right)^{\lambda\beta/\rho_r} \left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\lambda\beta/\rho_r}{1-\alpha_r}} \mathrm{d}X\right)^{1-\alpha_r}
\end{aligned}
$$

and so:

$$
\left(Y_{jt}/A_{jt}\right)^{1+\alpha_r\lambda\beta/\rho_r} = \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{1-\alpha_r} \left(A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r}.
$$

Introducing

$$
H_{jt} \equiv \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},
$$

we can simplify the previous expression as

$$
\begin{aligned}
\left(Y_{jt}/A_{jt}\right)^{1+\alpha_r\lambda\beta/\rho_r} &= \left(H_{jt}\right)^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \left(A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r}, \\
Y_{jt} &= \left(A_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \left(H_{jt}\right)^{1-\alpha_r}.
\end{aligned}
$$

Then, we can write the wage as

$$
\begin{aligned}
W_{jt}(X) &= C_r X^{\theta_j} A_{jt} \left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} \\
&= C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{(1+\alpha_r\lambda\beta/\rho_r)}}.
\end{aligned}
$$

Finally, we can write the efficiency units of labor as

$$
\begin{aligned}
L_{jt} &= \int X^{\theta_j} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} W_{jt}(X)^{\lambda\beta/\rho_r} \mathrm{d}X \\
&= \int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} \left(C_r H_{jt}^{-\alpha_r}\right)^{\lambda\beta/\rho_r} \left(A_{jt}\right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \mathrm{d}X \\
&= H_{jt}^{1+\alpha_r\lambda\beta/\rho_r-\alpha_r\lambda\beta/\rho_r} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= H_{jt} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}.
\end{aligned}
$$

□

Lemma 2 (Uniqueness of H_{jt}): *The firm- and time-specific equilibrium constants*

H_{jt} are uniquely defined.

PROOF: As we have established in Lemma 1, for firm j in market r, H_{jt} solves the following system:

$$H_{jt} = \Bigg[\int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'} \lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r}^{\lambda} H_{j't}^{-\alpha_{r} \lambda} \right)^{\beta/\rho_{r}} A_{j't}^{\frac{\lambda\beta/\rho_{r}}{1+\alpha_{r}\lambda\beta/\rho_{r}}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} N M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r \lambda\beta/\rho_r}},$$

where we have replaced $K_{rt}(X)$ and then $I_{rt}(X)$ and finally Y_{jt} with their expressions in terms of H_{jt}. We will show that $\tilde{H}_{jt} \equiv (H_{jt})^{\alpha_r}$ is unique, which implies that H_{jt} is unique. Defining $\vec{H}_t \equiv (\tilde{H}_{1t}, ..., \tilde{H}_{Jt})$, we will show that $\vec{H}_t$ solves the following fixed point expression:

$$\text{(4)} \quad \tilde{H}_{jt} = \Bigg[\int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r}} A_{j't}^{\frac{\lambda\beta/\rho_{r}}{1+\alpha_{r}\lambda\beta/\rho_{r}}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} N M(X) \mathrm{d}X \Bigg]^{\frac{\alpha_r}{1+\alpha_r \lambda\beta/\rho_r}}$$
$$= \Gamma_{jt}(\vec{H}_t).$$

We show that this expression satisfies the two conditions required to apply Theorem 1 of Kennan (2001). We first consider the component that is common to all j given by

$$\bar{\Gamma}_t(X, \vec{H}_t) \equiv \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$

and see that

$$\begin{aligned}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t) &= \left(\sum_{r'}\mu^{-\lambda\beta}\left(\sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_{r'}}A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}\right)^{\rho_{r'}}\right)^{-1}\\ &= \mu^{\lambda\beta}\bar{\Gamma}_t(X,\vec{H}_t).\end{aligned}$$

Hence

$$\begin{aligned}\Gamma_{jt}(\mu\cdot\vec{H}_t) =&\Bigg[\int X^{\theta_j(1+\lambda\beta/\rho_r)}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t)\left(\tau G_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\\ &\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)\mu^{-\lambda}C_r^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_r}A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1}NM(X)\mathrm{d}X\Bigg]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}}\\ =&\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\Bigg[\int X^{\theta_j(1+\beta/\rho_r)}\bar{\Gamma}_t(X,\vec{H}_t)\left(\tau G_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\\ &\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_r}A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1}NM(X)\mathrm{d}X\Bigg]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}}\\ =&\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\Gamma_{jt}(\vec{H}_t).\end{aligned}$$

Then for any $0<\mu<1$, r and $j\in J_r$, given $\vec{H}_t>0$ such that $\Gamma_t(\vec{H}_t)=\vec{H}_t$, where $\Gamma_t(\cdot)\equiv(\Gamma_{1t}(\cdot),...,\Gamma_{Jt}(\cdot))$, we have

$$\begin{aligned}\Gamma_{jt}(\mu\cdot\vec{H}_t)-\mu\cdot\tilde{H}_{jt} &= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\Gamma_{jt}(\vec{H}_t)-\mu\cdot\tilde{H}_{jt}\\ &= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\tilde{H}_{jt}-\mu\cdot\tilde{H}_{jt}\\ &= \mu\underbrace{(\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}-1}-1)}_{>0}\cdot\tilde{H}_{jt}\\ &> 0,\end{aligned}$$

which means that we have shown that $\Gamma_t(\vec{H}_t)-\vec{H}_t$ is strictly "radially quasi-concave". The next step is to show monotonicity. Consider $\vec{H}_{1t}$ and $\vec{H}_{2t}$ such that for a given j we have $\tilde{H}_{1jt}=\tilde{H}_{2jt}$ and $\tilde{H}_{1j't}\leq\tilde{H}_{2j't}$ for all other $j'\neq j$. Then we have that for all j', t, X and $r'=r(j')$,

$$\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}}A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}\geq\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}}A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}$$

and for any r',

$$\sum_{j'\in J_{r'}} \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \geq \sum_{j'\in J_{r'}} \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}.$$

Hence, summing over r' and taking it to the power of minus one, this implies that $\bar{\Gamma}_t(X,\vec{H}_{1t}) \leq \bar{\Gamma}_t(X,\vec{H}_{2t})$. Then, since $\rho_r \leq 1$ we also have that

$$\left(\sum_{j'\in J_r} \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} \leq \left(\sum_{j'\in J_r} \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1}.$$

Combining the last two results and observing that the third term in the expression for $\Gamma_{jt}(\vec{H}_t)$ is the same for $\vec{H}_{1t}$ and $\vec{H}_{2t}$ gives us that:

$$\Gamma_{jt}(\vec{H}_{1t}) \leq \Gamma_{jt}(\vec{H}_{2t}).$$

Then

$$\Gamma_{jt}(\vec{H}_{1t}) - \tilde{H}_{1j't} \leq \Gamma_{jt}(\vec{H}_{2t}) - \tilde{H}_{2j't},$$

and since the last inequality holds for all j, we obtain the quasi-increasing property:

$$\Gamma_{jt}(\vec{H}_{1t}) - \vec{H}_{1t} \leq \Gamma_{jt}(\vec{H}_{2t}) - \vec{H}_{2t}.$$

The fact that the function is "radially quasi-concave" together with monotonicity gives uniqueness of the fixed point by the theorem in Kennan (2001). This means that $\vec{H}_t$ is unique, and hence that $\tilde{H}_{jt}$ is unique and finally that H_{jt} is unique. $\square$

Definition 1: *We consider a sequence of increasingly larger economies indexed by an increasing number of regions n^r where $n_r^f = \kappa_r n^r$ for some fixed κ_r. In this sequence of economies we assume that the amenities scale according to $G_j(X) = \mathring{G}_j(X)\left(n_{r(j)}^f\right)^{-\rho_{r(j)}/\beta}$ for some fixed $\mathring{G}_j(X)$. We also assume that the mass of workers grows according to $N = n^r \cdot \bar{n}^f \cdot \mathring{N} = n^r \cdot n^r \cdot \bar{\kappa} \cdot \mathring{N}$, where $\bar{n}^f$ is the average of n_r^f and $\bar{\kappa}$ is the average of κ_r.*

Lemma 3: *The unique solution for H_{jt} in the limit of a sequence of growing economies is given by*

$$H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}},$$

where H_j solves the following fixed point:

$$H_j = \left(\int X^{\theta_j} \left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) dX \right)^{\frac{1}{1+\alpha_r \lambda\beta/\rho_r}}$$

$$I_{r0}(X)^{\lambda\beta/\rho_r} \equiv \mathbb{E}_j \left[\left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} H_j^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$

$$I_0(X)^{\lambda\beta} \equiv \mathbb{E}_r \left[I_{r0}(X)^{\lambda\beta} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right].$$

Proof: Consider the expression for H_{jt} from the beginning of Lemma 2:

$$H_{jt} = \Bigg[\int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} N M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.$$

We substitute in n^{r} , n_r^{f}, κ_r, $\mathring{G}_j(X) = \left(n_{r(j)}^{\mathrm{f}} \right)^{\rho_{r(j)}/\beta} G_j(X)$ and $\mathring{N} = (n^{\mathrm{r}}\ n^{\mathrm{r}}\ \bar{\kappa})^{-1} N$
:

$$H_{jt} = \Bigg[\int \left(\frac{1}{n^{\mathrm{r}}} \sum_{r'} \left(\frac{1}{n_{r'}^{\mathrm{f}}} \sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\frac{1}{n_r^{\mathrm{f}}} \sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_j(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.$$

As the economy grows large, i.e. as n^{r} grows to infinity, we have

$$H_{jt} = \Bigg[\int \Bigg(\mathbb{E}_{r'} \Bigg[\Bigg(\mathbb{E}_{j' \in J_{r'}} \Bigg[\left(X^{\lambda \theta_{j'}} \tau \mathring{G}_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \Bigg] \Bigg)^{\rho_{r'}} \Bigg] \Bigg)^{-1}$$
$$\times \left(\mathbb{E}_{j' \in J_r} \left[\left(X^{\lambda \theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r \lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \right)^{\rho_r - 1}$$
$$\times X^{\theta_j (1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.$$

Next we show that H_{jt} can indeed be expressed as stated in this Lemma. We guess that $H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r - 1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$ and verify that it solves the problem. To verify, note that

$$\mathbb{E}_{j' \in J_r} \left[\left(X^{\lambda \theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r \lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$
$$= \mathbb{E}_{j' \in J_r} \left[\left(X^{\lambda \theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j'}^{-\alpha_r \lambda} \right)^{\beta/\rho_r} \times \bar{A}_{rt}^{-\alpha_r\lambda\beta/\rho_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$
$$= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \mathbb{E}_{j' \in J_r} \left[\left(X^{\lambda \theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j'}^{-\alpha_r \lambda} \right)^{\beta/\rho_r} \tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$
$$= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r},$$

where we used $A_{jt} = \bar{A}_{r(j)t} \tilde{A}_{jt}$. Hence

$$H_{jt} = \Bigg[\int \left(\mathbb{E}_{r'} \left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta} \right] \right)^{-1} \times \left(\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$
$$= \left[\int X^{\theta_j} \left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$
$$\times \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta} \frac{\rho_r - 1}{1+\alpha_r\lambda\beta/\rho_r}}$$
$$= H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}},$$

where we used that H_j solves

$$H_j = \left[\int X^{\theta_j}\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

with

$$I_0(X) \equiv \left(\mathbb{E}_{r'}\left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta}\right]\right)^{1/(\lambda\beta)}.$$

□

We can then establish the final result.

Proposition 1: *The wage equation is given by*

$$w_j(x,\overline{a},\tilde{a}) = c_r + \theta_j x - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha_r\lambda\beta}\overline{a},$$

where

$$h_j = \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a}_{rt}.$$

Proof: Recall $L_{jt} = H_{jt}A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}$ from Lemma 1 and $H_{jt} = H_j\cdot\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$ from Lemma 3. Then:

$$\begin{aligned}
h_{jt} &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}a_{jt} \\
&= \frac{(\rho_r-1)\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)(1+\alpha_r\lambda\beta/\rho_r)}\overline{a}_{rt} + h_j.
\end{aligned}$$

Hence, we get

$$\begin{aligned}
h_j &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a}_{rt} \\
\ell_{jt} &= h_j + \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} + \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a}_{rt} \\
&\equiv \ell_j(\overline{a}_{rt},\tilde{a}_{jt}).
\end{aligned}$$

Next, we replace H_{jt} and A_{jt} in the expression for the wage from Lemma 1, $W_{jt}(X) = C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$ to get

$$\begin{aligned}
w_{jt}(x) &= c_r + \theta_j x - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} + \frac{1}{1+\alpha_r\lambda\beta}\overline{a}_{rt} \\
&\equiv w_j(x,\overline{a}_{rt},\tilde{a}_{jt}).
\end{aligned}$$

Note that $w_{jt}(x)$ depends on time only through $\overline{a}_{rt}$ and $\tilde{a}_{jt}$.

$\square$

COROLLARY 1: *The firm's demand for labor is given by:*

$$D_{jt}(X) = \frac{N}{n^r} M(X) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda} W_{jt}(X)}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}} \right)^{\lambda\beta/\rho_r}.$$

PROOF: As n^{r} grows to infinity, we first note:

$$\begin{aligned}
I_{rt}(X)^{\lambda\beta/\rho_r} &= \sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} C_r X^{\theta_{j'}} A_{j't} \left(\frac{Y_{j't}}{A_{j't}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} \right)^{\lambda\beta/\rho_r} \\
&= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \frac{1}{n_r^{\mathrm{f}}} \sum_{j' \in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j'}^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\
&= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r} \\
I_{rt}(X) &= \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}} I_{r0}(X).
\end{aligned}$$

The firm's demand can then be written as:

$$\begin{aligned}
D_{jt}(X) &= N M(X) \frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}} \left(G_j(X)^{1/\lambda} \tau^{1/\lambda} \frac{W_{jt}(X)}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} \\
&= \frac{N}{n^{\mathrm{r}}} M(X) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda} W_{jt}(X)}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}} \right)^{\lambda\beta/\rho_r}.
\end{aligned}$$

$\square$

We also derive the other quantities of the model.

COROLLARY 2: *The firm's value added and wage bill are given by*

$$\begin{aligned}
y_j(\overline{a}, \tilde{a}) =& (1-\alpha_r) h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta} \overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a} \\
b_j(\overline{a}, \tilde{a}) =& c_r + (1-\alpha_r) h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta} \overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}.
\end{aligned}$$

Proof: For the firm's value added, note that

$$
\begin{aligned}
Y_{jt} &= H_{jt}^{1-\alpha_r} A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}\right)^{1-\alpha_r} \left(\bar{A}_{rt}\tilde{A}_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
y_{jt} &= (1-\alpha_r)h_j + \left(\frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\right)\overline{a}_{rt} + \left(\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\right)\tilde{a}_{jt} \\
&\equiv y_j(\overline{a}_{rt}, \tilde{a}_{jt})
\end{aligned}
$$

and for the wage bill,

$$
\begin{aligned}
B_{jt} &= \int W_{jt}(X)D_{jt}(X)\mathrm{d}X \\
&= \int W_{jt}(X)\left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r}(W_{jt}(X))^{\lambda\beta/\rho_r}\frac{NM(X)}{n^{\mathrm{r}}}\mathrm{d}X \\
&= \int \left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r} \\
&\qquad \times \left(C_r X^{\theta_j}H_j^{-\alpha_r}\right)^{1+\lambda\beta/\rho_r}\left(\tilde{A}_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\bar{A}_{rt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}n^{\mathrm{r}}\bar{\kappa}\mathring{N}M(X)\mathrm{d}X \\
b_{jt} &= c_r + (1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha\beta}\overline{a}_{rt} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} \\
&= b_j(\overline{a}_{rt}, \tilde{a}_{jt}).
\end{aligned}
$$

It follows that

$$
y_j(\overline{a}, \tilde{a}) - b_j(\overline{a}, \tilde{a}) = c_r.
$$

$\square$

Note that the previous expressions deliver the structural pass-through rates of market and firm level shocks (with abuse of notation):

$$
\begin{aligned}
\frac{\partial w_j(x, \overline{a}_{rt}, \tilde{a}_{jt})}{\partial \overline{a}} \cdot \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \overline{a}}\right)^{-1} &= \frac{1}{1+\lambda\beta} \\
\frac{\partial w_j(x, \overline{a}_{rt}, \tilde{a}_{jt})}{\partial \tilde{a}} \cdot \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \tilde{a}}\right)^{-1} &= \frac{\rho_r}{\rho_r+\lambda\beta}.
\end{aligned}
$$

Corollary 3: *Firm j worker composition does not depend on $\overline{a}$ or $\tilde{a}$.*

PROOF: Consider $\Pr[X|j,t]$:

$$
\begin{aligned}
\Pr[X|j,t] &= \Pr[X,j|t]/\Pr[j|t] \\
&= \frac{\Pr[j|X,t]\Pr[X]}{\int \Pr[j|X',t]M(X')\mathrm{d}X'} \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(\tau\mathring{G}_j(X)W_{jt}(X)^{\lambda}\right)^{\beta/\rho_r} M(X)}{\int\left(\frac{I_{r0}(X')}{I_0(X')}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X')}\right)^{\lambda\beta/\rho_r}\left(\tau\mathring{G}_j(X')W_{jt}(X')^{\lambda}\right)^{\beta/\rho_r} M(X')\mathrm{d}X'} \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\mathring{G}_j(X)\right)^{\beta/\rho_r} M(X)}{\int\left(\frac{I_{r0}(X')}{I_0(X')}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X')}\right)^{\lambda\beta/\rho_r}\left(X'^{\lambda\theta_j}\mathring{G}(X')\right)^{\beta/\rho_r} M(X')\mathrm{d}X'} \\
&= \Pr[X|j].
\end{aligned}
$$

where we used the fact that

$$
\begin{aligned}
\Pr[j|X,t] &= D_{jt}(X)/M(X) \\
&= \frac{N}{n^{\mathrm{r}}}\left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}W_{jt}(X)}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r}.
\end{aligned}
$$

□

2. Worker rents

LEMMA 4: *We establish that for workers of type X working at firm j in market r at time t, the average firm-level rents are given by $\frac{W_{jt}(X)}{1+\lambda\beta/\rho_r}$ and the average market level rents are given by $\frac{W_{jt}(X)}{1+\lambda\beta}$.*

PROOF: The average worker rents at the firm are defined as the difference between the worker's willingness to accept W and the wage they actually get at firm j at time t, denoted by $W_{jt}(X)$. The supply curve $S_{jt}(X,W)$ exactly defines the number of people willing to work at firm j at some given wage W. Hence, the density of the willingness to accept among workers in firm j at time t at wage $W_{jt}(X)$ is given by:

$$\frac{1}{S_{jt}(X,W_{jt}(X))}\frac{\partial S_{jt}(X,W)}{\partial W}.$$

We obtain the average rents by taking the expectation with respect to this density:

$$
\begin{aligned}
R^w_{jt}(X) &\equiv \mathbb{E}\left[R^w_{it} \,|\, j(i,t)=j, X_i = X\right] \\
&= \int_0^{W_{jt}(X)} \left(W_{jt}(X) - W\right) \frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial S_{jt}(X,W)}{\partial W} \mathrm{d}W \\
&= W_{jt}(X) \int_0^1 (1-\omega) \frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial S_{jt}(X, \omega W_{jt}(X))}{\partial \omega} \mathrm{d}\omega \\
&= W_{jt}(X) \int_0^1 (1-\omega) \frac{\partial \omega^{\lambda\beta/\rho_r}}{\partial \omega} \mathrm{d}\omega \\
&= \frac{W_{jt}(X)}{1+\lambda\beta/\rho_r},
\end{aligned}
$$

where the second to last step relies on the definition of $S_{jt}(X,W)$ and the fact that we assume the presence of many firms in each market to show that $S_{jt}(X,\omega W) = \omega^{\lambda\beta/\rho_r} S_{jt}(X,W)$. We can then take the average over the productivity levels X_i of the workers i in firm $j \in J_r$ at time t to get:

$$
\begin{aligned}
\mathbb{E}\left[R^w_{it} \,|\, j(i,t)=j\right] &= \mathbb{E}\left[R^w_{jt}(X_i) \,|\, j(i,t)=j\right] \\
&= \frac{1}{1+\lambda\beta/\rho_r} \mathbb{E}\left[W_{jt}(X_i) \,|\, j(i,t)=j\right].
\end{aligned}
$$

Next we want to compute the integral of the market-level supply curve for each worker of type X. In contrast to the worker rents at the firm level, we want to shift the wages of all firms in a given market for a given individual. This means that we want to shift both the current firm j but also all other firms j' in market r. Given the labor supply curve of firm j, we integrate by scaling all wages in market r by ω in $[0,1]$. More precisely, we consider the demand realized by the set of wages $\left\{\omega^{\mathbf{1}[j\in J_r]} W_{jt}(X)\right\}_{jt}$ for a given market r. The supply curve of firm j in this market as a function of the scaling factor ω is then

$$
\begin{aligned}
& N \cdot M(X) \frac{\left(\sum_{j'\in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j'\in J_{r'}} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} W_{j't}(X)\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \\
&\qquad \times \frac{\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{jt}(X)\right)^{\lambda\beta/\rho_r}}{\sum_{j'\in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X)\right)^{\lambda\beta/\rho_r}} \\
&= \omega^{\lambda\beta} S_{jt}(X, W_{jt}(X)),
\end{aligned}
$$

where we used the assumption that there are many markets in the first denominator. Hence, the market level density of the willingness to accept is given by

$$\frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial}{\partial \omega} \left[\omega^{\lambda\beta} S_{jt}(X, W_{jt}(X)) \right].$$

Using the same logic we used to solve for the firm level rents, we find

$$\begin{aligned} R^{wm}_{jt}(X) &\equiv \mathbb{E}\left[R^{wm}_{it} \,|\, j(i,t) = j, X_i = X \right] \\ &= \frac{W_{jt}(X)}{1 + \lambda\beta}, \end{aligned}$$

and can finally compute the average market level rents across X_i as

$$\begin{aligned} \mathbb{E}\left[R^{wm}_{it} \,|\, j(i,t) = j \right] &= \mathbb{E}\left[R^{wm}_{jt}(X_i) \,|\, j(i,t) = j \right] \\ &= \frac{1}{1 + \lambda\beta} \mathbb{E}\left[W_{jt}(X_i) \,|\, j(i,t) = j \right]. \end{aligned}$$

□

3. Employer rents

LEMMA 5: *We establish that the firm rents are given by*

$$R^f_{jt} = \Pi_{jt} - \Pi^{pt}_{jt} \; = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right) \Pi_{jt}.$$

PROOF: The firm rents are defined as the difference between the profit that a firm would make if it were a wage taker in the labor market and the profit it actually achieves when taking advantage of its wage setting power. To solve for the wage taker profit, we maximize

$$\Pi^{\mathrm{pt}}_{jt} \; = \max_{\{D^{\mathrm{pt}}_{jt}(X)\}} A_{jt} \left(\int X^{\theta_j} \cdot D^{\mathrm{pt}}_{jt}(X) \mathrm{d}X \right)^{1-\alpha_r} - \int W^{\mathrm{pt}}_{jt}(X) \cdot D^{\mathrm{pt}}_{jt}(X) \mathrm{d}X,$$

taking the wage $W^{\mathrm{pt}}_{jt}(X)$ as given, and then equate demand with the supply equation. The first order condition is

$$\underbrace{(1-\alpha_r)}_{\equiv C^{\mathrm{pt}}_r} A_{jt} X^{\theta_j} \left(\frac{Y^{\mathrm{pt}}_{jt}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} = W^{\mathrm{pt}}_{jt} \, (X)$$

and the realized demand is given by

$$D_{jt}^{\mathrm{pt}}(X) = N \cdot M(X) \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} \left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \frac{W_{jt}^{\mathrm{pt}}\ (X)}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r},$$

where we use $I(X)^{\lambda\beta} \equiv \sum_{r'} I_{r't}(X)^{\lambda\beta}$, assumed constant due to the large number of markets. We then get that

$$\begin{aligned}
\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} &= \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X \right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}^{\mathrm{pt}}\ (X)}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\mathrm{pt}} A_{jt} X^{\theta_j}}{I_{rt}(X)} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{1-\alpha_r} \\
&= (A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C_r^{\mathrm{pt}}}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C_r}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{1-\alpha_r}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{1+\alpha_r\lambda\beta/\rho_r} &= \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{(1-\alpha_r)\lambda\beta/\rho_r} H_{jt}^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \\
Y_{jt}^{\mathrm{pt}} &= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r\lambda\beta/\rho_r}} Y_{jt},
\end{aligned}$$

which we replace to get the wage

$$
\begin{aligned}
W_{jt}^{\mathrm{pt}}(X) &= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} \\
&= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot C_r A_{jt} X^{\theta_j} \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X).
\end{aligned}
$$

Similarly, we can express demand as

$$
D_{jt}^{\mathrm{pt}}(X) = \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X)
$$

and the wage bill as

$$
\begin{aligned}
B_{jt}^{\mathrm{pt}} &= \int W_{jt}^{\mathrm{pt}}(X) \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X \\
&= \int \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X) \cdot \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X) \mathrm{d}X \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} B_{jt}.
\end{aligned}
$$

Next, we recall $Y_{jt} = A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt}^{1-\alpha_r}$ and get that:

$$
\begin{aligned}
B_{jt} &= \int W_{jt}(X) \cdot D_{jt}(X) \mathrm{d}X \\
&= \int X^{\theta_j} C_r H_{jt}^{-\alpha_r} \left(A_{jt}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot D_{jt}(X) \mathrm{d}X \\
&= C_r H_{jt}^{-\alpha_r} \left(A_{jt}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \left(\frac{Y_{jt}}{A_{jt}}\right)^{\frac{1}{1-\alpha_r}} \\
&= C_r H_{jt}^{-\alpha_r} \left(A_{jt}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt} \left(A_{jt}\right)^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} \\
&= C_r Y_{jt}.
\end{aligned}
$$

Similarly, we get that $B_{jt}^{\mathrm{pt}} = C_r^{\mathrm{pt}} Y_{jt}^{\mathrm{pt}}$. Finally, we see that

$$
\begin{aligned}
\frac{\Pi_{jt} - \Pi_{jt}^{\mathrm{pt}}}{\Pi_{jt}} &= 1 - \frac{Y_{jt}^{\mathrm{pt}} - B_{jt}^{\mathrm{pt}}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C_r^{\mathrm{pt}}}{1 - C_r} \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= 1 - \frac{\alpha_r \left(1+\lambda\beta/\rho_r\right)}{1+\alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}
\end{aligned}
$$

$$
\Pi_{jt} - \Pi_{jt}^{\mathrm{pt}} = \left(1 - \frac{\alpha_r \left(1+\lambda\beta/\rho_r\right)}{1+\alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right) \Pi_{jt}.
$$

$\square$

LEMMA 6: *We establish that the market level rents for firm $j \in J_r$ are given by*

$$
R_{jt}^{fm} = \Pi_{jt} - \Pi_{jt}^{ptm} = \left(1 - \frac{\alpha_r \left(1+\lambda\beta/\rho_r\right)}{1+\alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}\right) \Pi_{jt}.
$$

PROOF: Here we consider the case where all firms in a given market are wage takers. In this case we also get that the $I_{rt}(X)$ terms change. The firm's wage is still determined by the following first-order condition:

$$
(1-\alpha_r) A_{jt} X^{\theta_j} \left(\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} = W_{jt}^{\mathrm{ptm}}(X).
$$

However, the labor supply curve is no longer the same as in equilibrium since all firms change their labor demands:

$$S_{jt}^{\mathrm{ptm}}(X,W) = NM(X)\left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)}\right)^{\lambda\beta}\left(G_j(X)^{1/\lambda}\frac{\tau^{1/\lambda}W}{I_{rt}^{\mathrm{ptm}}(X)}\right)^{\lambda\beta/\rho_r},$$

where

$$I_{rt}^{\mathrm{ptm}}(X) \equiv \left(\sum_{j'\in J_r}\left(\tau G_{j'}(X)\right)^{\beta/\rho_r}\left(W_{j't}^{\mathrm{ptm}}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)}.$$

We insert these definitions into Y_{jt}^{ptm} to see that

$$\begin{aligned}
\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}} &= \left(\int X^{\theta_j}\cdot D_{jt}^{\mathrm{ptm}}(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j}\cdot\left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)}\right)^{\lambda\beta}(\tau G_j(X))^{\beta/\rho_r}\left(\frac{W_{jt}^{\mathrm{ptm}}(X)}{I_{rt}^{\mathrm{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j}\cdot\left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)}\right)^{\lambda\beta}(\tau G_j(X))^{\beta/\rho_r}\left(\frac{(1-\alpha_r)A_{jt}X^{\theta_j}}{I_{rt}^{\mathrm{ptm}}(X)}\left(\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-} \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r}\underbrace{\left(\int X^{\theta_j}\cdot\left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)}\right)^{\lambda\beta}(\tau G_j(X))^{\beta/\rho_r}\left(\frac{C_r^{\mathrm{pt}}X^{\theta_j}}{I_{rt}^{\mathrm{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)}_{\equiv\left(H_{jt}^{\mathrm{ptm}}\right)^{1+\alpha_r\lambda\beta/\rho_r}}{}^{1-\alpha_r}\left(\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}}\right. \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r}\left(H_{jt}^{\mathrm{ptm}}\right)^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)}\left(\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
\frac{Y_{jt}^{\mathrm{ptm}}}{A_{jt}} &= A_{jt}^{\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\left(H_{jt}^{\mathrm{ptm}}\right)^{1-\alpha_r} \\
Y_{jt}^{\mathrm{ptm}} &= A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\left(H_{jt}^{\mathrm{ptm}}\right)^{1-\alpha_r}
\end{aligned}$$

This allows us to write the wage equation as

$$W_{jt}^{\mathrm{ptm}}(X) = C_r^{\mathrm{pt}}X^{\theta_j}\left(H_{jt}^{\mathrm{ptm}}\right)^{-\alpha_r}A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.$$

As in the baseline equilibrium, we are left with finding H_{jt}^{ptm} as a function of the market TFP and amenities:

$$H_{jt}^{\text{ptm}} = \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}} \; X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} .$$

Note that

$$\begin{aligned}
I_{rt}^{\text{ptm}}(X) &= \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(W_{j't}^{\text{ptm}}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r/(\lambda\beta)} \\
&= \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r^{\text{pt}} \; X^{\theta_{j'}} \left(H_{j't}^{\text{ptm}} \right)^{-\alpha_r} \right)^{\lambda\beta/\rho_r} (A_{j't})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r/(\lambda\beta)} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r} \right) \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r X^{\theta_{j'}} \left(H_{j't}^{\text{ptm}} \right)^{-\alpha_r} \right)^{\lambda\beta/\rho_r} (A_{j't})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r/(\lambda\beta)}
\end{aligned}$$

We want to show that $H_{jt}^{\text{ptm}} = \left(\frac{C_r^{\text{pt}}}{C_r} \right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta} 1[j \in J_r]} H_{jt}$. To see this we observe that $\tilde{H}_{jt}^{\text{ptm}}$ solves a very similar fixed point to $\tilde{H}_{jt}$. Indeed

$$\begin{aligned}
\tilde{H}_{jt}^{\text{ptm}} &= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}} \; X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)} \right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X \right)^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r} \right)^{\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r} 1[j \in J_r]} \Gamma_{jt} \left[\vec{H}_t^{\text{ptm}} \right],
\end{aligned}$$

where $\Gamma_{jt}(\cdot)$ is the operator defined in Lemma 2, equation (19) that defines H_{jt} as a fixed point. For this operator, we know that $\Gamma_{jt}(\vec{H}_t) = \tilde{H}_{jt}$ is the unique fixed point. The next step is to check that $\vec{H}_t'$, defined such that its j component,

$\tilde{H}'_{jt} = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}}$, is a fixed point of the same operator $\Gamma_{jt}(\cdot)$:

$$\begin{aligned}
\Gamma_{jt}\left(\vec{H}'_t\right) &= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \Gamma_{jt}\left(\vec{H}_t^{\text{ptm}}\right) \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \\
&= \tilde{H}'_{jt},
\end{aligned}$$

hence $H'_{jt} = H_{jt}$ for all j and so we get that

$$H_{jt}^{\text{ptm}} = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}.$$

So, for $j \in J_r$, we find that

$$\begin{aligned}
W_{jt}^{\text{ptm}}(X) &= C_r^{\text{pt}}\, X^{\theta_j} \left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= C_r^{\text{pt}}\, X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta}} W_{jt}(X)
\end{aligned}$$

and then

$$\begin{aligned}
I_{rt}^{\text{ptm}}(X) &= \left(\sum_{j'\in J_r} \left(\tau G_{j'}(X)\right)^{\beta/\rho_r} \left(W_{j't}^{\text{ptm}}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta}} I_{rt}(X).
\end{aligned}$$

Next, let us rewrite the realized demand:

$$
\begin{aligned}
D_{jt}^{\text{ptm}}(X) &= \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}^{\text{ptm}}(X)}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} D_{jt}(X).
\end{aligned}
$$

We can then compute the firm's output and wage bill:

$$
\begin{aligned}
Y_{jt}^{\text{ptm}} &= A_{jt}\left(\int X^{\theta_j} \cdot D_{jt}^{\text{ptm}}(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} Y_{jt} \\
B_{jt}^{\text{ptm}} &= \int W_{jt}^{\text{ptm}}(X) \cdot D_{jt}^{\text{ptm}}(X)\mathrm{d}X \\
&= C_r^{\text{pt}}\, Y_{jt}^{\text{ptm}}.
\end{aligned}
$$

Finally, we establish that:

$$
\begin{aligned}
\frac{\Pi_{jt} - \Pi_{jt}^{\text{ptm}}}{\Pi_{jt}} &= 1 - \frac{Y_{jt}^{\text{ptm}} - B_{jt}^{\text{ptm}}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C_r^{\text{pt}}}{1 - C_r}\left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \\
&= 1 - \frac{\alpha_r\,(1+\lambda\beta/\rho_r)}{1+\alpha_r\lambda\beta/\rho_r}\left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}
\end{aligned}
$$

$$
\Pi_{jt} - \Pi_{jt}^{\text{ptm}} = \left(1 - \frac{\alpha_r\,(1+\lambda\beta/\rho_r)}{1+\alpha_r\lambda\beta/\rho_r}\left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}\right)\Pi_{jt}.
$$

□

4. *Walrasian equilibrium, wedges, tax policy, and welfare*

Walrasian equilibrium

We consider an equilibrium as defined by a set of wages $W^{\mathrm{c}}_{jt}(X)$ such that workers optimally choose where to work given these wages, and firms optimally choose labor demand, also taking these wages as given. In this equilibrium we make the tax system neutral $\lambda = \tau = 1$:

$$\max_{\{D^{\mathrm{c}}_{jt}(X)\}} A_{jt}\left(\int X^{\theta_j} D^{\mathrm{c}}_{jt}(X)\mathrm{d}X\right)^{1-\alpha_r} - \int W^{\mathrm{c}}_{jt}(X)D^{\mathrm{c}}_{jt}(X)\mathrm{d}X,$$

which gives the first order condition

$$(1-\alpha_r)X^{\theta_j}A_{jt}\left(\int X^{\theta_j}\cdot D^{\mathrm{c}}_{jt}(X)\mathrm{d}X\right)^{-\alpha_r} = W^{\mathrm{c}}_{jt}(X)$$

or

$$W^{\mathrm{c}}_{jt}(X) = \underbrace{(1-\alpha_r)}_{\equiv C^{\mathrm{pt}}_r} X^{\theta_j}A_{jt}\left(\frac{Y^{\mathrm{c}}_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}.$$

We then solve for output

$$\begin{aligned}
\left(\frac{Y^{\mathrm{c}}_{jt}}{A_{jt}}\right)^{\frac{1}{1-\alpha_r}} &= \int X^{\theta_j}\cdot D^{\mathrm{c}}_{jt}(X)\mathrm{d}X \\
&= \int X^{\theta_j}NM(X)\cdot\frac{(I^{\mathrm{c}}_{rt}(X))^{\beta}}{\sum_{r'}\left(I^{\mathrm{c}}_{r't}(X)\right)^{\beta}}\cdot\left(\frac{W^{\mathrm{c}}_{jt}(X)G_j(X)}{I^{\mathrm{c}}_{rt}(X)}\right)^{\beta/\rho_r}\mathrm{d}X \\
&= \int X^{\theta_j}\cdot\frac{(I^{\mathrm{c}}_{rt}(X))^{\beta}}{\sum_{r'}\left(I^{\mathrm{c}}_{r't}(X)\right)^{\beta}}\cdot\left(\frac{C^{\mathrm{pt}}_r X^{\theta_j}G_j(X)}{I^{\mathrm{c}}_{rt}(X)}\right)^{\beta/\rho_r} NM(X)\mathrm{d}X\times A_{jt}^{\beta/\rho_r}\left(\frac{Y^{\mathrm{c}}_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\beta/\rho_r}{1-\alpha_r}} \\
&= \left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha_r\beta/\rho_r} A_{jt}^{\beta/\rho_r}\left(\frac{Y^{\mathrm{c}}_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\beta/\rho_r}{1-\alpha_r}} \\
\frac{Y^{\mathrm{c}}_{jt}}{A_{jt}} &= A_{jt}^{\frac{(1-\alpha_r)\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\left(H^{\mathrm{c}}_{jt}\right)^{1-\alpha_r} \\
Y^{\mathrm{c}}_{jt} &= A_{jt}^{\frac{1+\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\left(H^{\mathrm{c}}_{jt}\right)^{1-\alpha_r},
\end{aligned}$$

where we defined

$$\left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha\beta/\rho_r} \equiv \int X^{\theta_j}\cdot\frac{(I^{\mathrm{c}}_{rt}(X))^{\beta}}{\sum_{r'}\left(I^{\mathrm{c}}_{r't}(X)\right)^{\beta}}\cdot\left(\frac{C^{\mathrm{pt}}_r X^{\theta_j}G_j(X)}{I^{\mathrm{c}}_{rt}(X)}\right)^{\beta/\rho_r} NM(X)\mathrm{d}X,$$

giving the wage:

$$W^{\mathrm{c}}_{jt}(X) = C^{\mathrm{pt}}_r X^{\theta_j} \left(H^{\mathrm{c}}_{jt}\right)^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\beta/\rho_r}} .$$

Next, using $H^{\mathrm{c}}_{jt} = H^{\mathrm{c}}_j \bar{A}_{rt}^{\frac{(\rho_r-1)\beta/\rho_r}{(1+\alpha_r\beta)(1+\alpha_r\beta/\rho_r)}}$ and following a similar proof to the main proposition we find that

$$w^{\mathrm{c}}_j(x, \overline{a}, \tilde{a}) = c^{\mathrm{pt}} + \theta_j x - \alpha_r h^{\mathrm{c}}_j + \frac{1}{1+\alpha_r\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha_r\beta}\overline{a},$$

where

$$H^{\mathrm{c}}_j = \left[\int X^{\theta_j(1+\beta/\rho_r)} \left(\frac{I^{\mathrm{c}}_{r0}(X)}{I^{\mathrm{c}}_0(X)}\right)^{\beta} \left(\frac{1}{I^{\mathrm{c}}_{r0}(X)}\right)^{\beta/\rho_r} \left(C^{\mathrm{pt}}_r \mathring{G}_j(X)\right)^{\beta/\rho_r} \frac{\overline{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X\right]^{\frac{1}{1+\alpha_r\beta/\rho_r}}$$

$$I^{\mathrm{c}}_{r0}(X) = \left(\mathbb{E}_{j\in J_r}\left[\left(\mathring{G}_j(X) X^{\theta_j} C^{\mathrm{pt}}_r \left(H^{\mathrm{c}}_j\right)^{-\alpha_r}\right)^{\beta/\rho_r} \left(\tilde{A}_{jt}\right)^{\frac{\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\right]\right)^{\rho_r/\beta}$$

$$I^{\mathrm{c}}_0(X) = \left(\mathbb{E}_r\left[I^{\mathrm{c}}_{r0}(X)^{\beta} \left(\overline{A}_{rt}\right)^{\frac{\beta}{1+\alpha_r\beta}}\right]\right)^{1/\beta} .$$

We can then get the allocation of workers to each firm given by

$$\text{for } j \in J_r \qquad D^{\mathrm{c}}_{jt}(X) = n^{\mathrm{r}} \overline{\kappa} \mathring{N} M(X) \left(\frac{I^{\mathrm{c}}_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}}{I^{\mathrm{c}}_0(X)}\right)^{\beta} \left(\frac{G_j(X) W^{\mathrm{c}}_{jt}(X)}{I^{\mathrm{c}}_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}}\right)^{\beta/\rho_r} .$$

Defining wedges

To define wedges, we look at the decisions of firms to set wages, the decisions of workers to choose markets, and the decisions of workers to choose particular firms within a given market. We express each of these decisions in the monopsonistic competition model, clarifying where the sources of wedges are in each equation.

The first wedge is a productivity wedge reflected in the wage equation:

$$W_{jt}(X) = \Big(\underbrace{1 + \frac{\rho_r}{\lambda\beta}}_{\text{labor prod. wedge}} \Big)^{-1} \cdot \underbrace{X^{\theta_j}(1-\alpha_r) A_{jt} L_{jt}^{-\alpha_r}}_{\text{marginal product of labor: } \mathcal{M}_{jt}(X)} .$$

We next turn to the expression for the quantity of labor $D_{jt}(X)$. For this we compute the log odds ratio of choosing one firm j versus another firm j' within a market r. We have

$$\log \frac{\Pr[j(i,t){=}j|X_i{=}X,\mathbf{W}_t, j\in J_r]}{\Pr[j(i,t){=}j'|X_i{=}X,\mathbf{W}_t, j'\in J_r]} = \frac{\beta}{\rho_r}\left[\log\frac{G_j(X)}{G_{j'}(X)} + \underbrace{\lambda}_{\text{pref. wedge}} \log\frac{W_{jt}(X)}{W_{j't}(X)}\right],$$

where the allocation is identical in all respects aside from the presence of the tax parameter λ which acts as a preference wedge between amenities and earnings.

We now shift attention to how the worker chooses between two different markets $r \neq r'$. It is useful to express wages using the wage index $I_{rt}(X)$ from equation (2) to see that

$$\log \frac{\Pr[j(i,t)\in J_r|X_i{=}X,\mathbf{W}_t]}{\Pr[j(i,t)\in J_{r'}|X_i{=}X,\mathbf{W}_t]} = \underbrace{\lambda}_{\text{pref. wedge}} \beta\log\frac{I_{rt}(X)}{I_{r't}(X)}.$$

The results clarify two wedges: a productivity wedge equal to $1+\frac{\rho_r}{\lambda\beta}$ and a preference wedge equal to λ.

Defining tax policy counterfactuals

Lemma 7: *Setting a tax policy with* $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ *and* $\lambda = 1$ *achieves the competitive allocation of workers to firms.*

Proof: We substitute $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ into the firm's problem and show that it achieves the planner's solution in this context. Recall from Lemma 3

$$H_j = \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(\mathring{G}_j(X)\tau C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\overline{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

$$I_{r0}(X)^{\lambda\beta/\rho_r} = \mathbb{E}_j\left[\left(\tau\mathring{G}_j(X)X^{\lambda\theta_j}C_r^{\lambda}H_j^{-\lambda\alpha_r}\right)^{\beta/\rho_r}\tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right]$$

$$I_0(X)^{\lambda\beta} = \mathbb{E}_r\left[I_{r0}(X)^{\lambda\beta}\bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}}\right],$$

where we notice that τC_r^{λ} always appears together and under this particular policy we get that $\tau_r C_r^{\lambda} = (1-\alpha_r) = C_r^{\mathrm{pt}}$. Hence, h_j coincides exactly with h_j^{c} while $I_{r0}(X)$ and $I_0(X)$ coincide with $I_{r0}^{\mathrm{c}}(X)$ and $I_0^{\mathrm{c}}(X)$, respectively. We then see that this implies that $D_{jt}(X) = D_{jt}^{\mathrm{c}}(X)$. In other words such policy achieves exactly the planner's allocation. □

Defining welfare

We start by defining a measure of welfare given a set of wages and tax parameters. Recall that the average utility that a worker enjoys for a given set of wages is given by:

$$\mathbb{E}\left[u_{it}|\mathbf{W}_t\right] = \int \frac{1}{\beta}\left[\log\left(\sum_r \left(\sum_{j\in J_r} \left(\tau G_j(X)\right)^{\beta/\rho_r}\left(W_{jt}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r} + \overline{C}\right)\right] M(X)\mathrm{d}X,$$

where we normalize $\overline{C}$ to zero. The total tax revenue R_t and total firm profits Π_t are given by:

$$\begin{aligned}
R_t &= \int \sum_r \sum_{j\in J_r} D_{jt}(X)\left(W_{jt}(X) - \tau W_{jt}(X)^{\lambda}\right)\mathrm{d}X \\
&= \int \sum_r \sum_{j\in J_r} D_{jt}(X) W_{jt}(X)\mathrm{d}X - \int \sum_r \sum_{j\in J_r} D_{jt}(X)\tau W_{jt}(X)^{\lambda}\mathrm{d}X \\
&= B_t - B_t^{\text{net}} \\
\Pi_t &= \sum_r \sum_{j\in J_r} A_{jt}\left(\int X^{\theta_j}\cdot D_{jt}(X)\mathrm{d}X\right)^{1-\alpha_r} - \int W_{jt}(X)\cdot D_{jt}(X)\mathrm{d}X \\
&= Y_t - B_t.
\end{aligned}$$

To take into account changes in tax revenue and firm profits across counterfactuals, we redistribute Π_t and R_t to workers in the form of a non-distortionary payment proportional to their net wages, governed by ϕ_t. This means that each worker receives $\phi_t \tau W_{jt}(X)^{\lambda}$ in transfers. The total transfer equals $\Pi_t + R_t$ and is given by

$$\begin{aligned}
\int \sum_r \sum_{j\in J_r} \phi_t \tau W_{jt}(X)^{\lambda}\cdot D_{jt}(X)\mathrm{d}X &= \Pi_t + R_t \\
\phi_t B_t^{\text{net}} &= \Pi_t + R_t
\end{aligned}$$

which implies

$$\begin{aligned}
1+\phi_t &= \frac{\Pi_t + R_t + B_t^{\text{net}}}{B_t^{\text{net}}} \\
&= \frac{\Pi_t + B_t}{B_t^{\text{net}}} \\
&= \frac{Y_t}{B_t^{\text{net}}}.
\end{aligned}$$

Thus, welfare can be decomposed as

$$\mathbb{W}_t = \int \frac{1}{\beta}\left[\log \sum_r \left(\sum_{j\in J_r} ((1+\phi_t)\tau G_j(X))^{\beta/\rho_r} (W_{jt}(X))^{\lambda\beta/\rho_r}\right)^{\rho_r}\right] M(X)\mathrm{d}X$$
$$= \underbrace{\mathbb{E}\left[u_{it}|\mathbf{W}_t\right]}_{\text{utility from net-wages and amenities}} + \underbrace{\log(1+\phi_t).}_{\text{utility from redistributed profits and tax revenue}}$$

5. An extension with amenity shocks

LEMMA 8: *The unique solution for $\check{H}_{jt}$ in the limit of a sequence of growing economies with $G_{jt}(X) = \overline{G}_{rt}\tilde{G}_{jt}G_j(X)$ is given by*

$$\check{H}_{jt} = \check{H}_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} \cdot \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \cdot \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}},$$

where $\check{H}_j$ solves the following fixed point:

$$\check{H}_j = \left(\int X^{\theta_j}\left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)}\right)^{\lambda\beta}\left(\frac{1}{\check{I}_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)dX\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

$$\check{I}_{r0}(X)^{\lambda\beta/\rho_r} \equiv \mathbb{E}_j\left[\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\check{H}_j^{-\lambda\alpha_r}\right)^{\beta/\rho_r}\tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right]$$

$$\check{I}_0(X)^{\lambda\beta} \equiv \mathbb{E}_r\left[\check{I}_{r0}(X)^{\lambda\beta}\overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}}\bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}}\right].$$

PROOF: Consider the expression for H_{jt} from Lemma 3. Substitute in n^{r} , n_r^{f}, κ_r, $G_{jt}(X) = \overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X)\left(n_{r(j)}^{\mathrm{f}}\right)^{-\rho_{r(j)}/\beta}$ and $\mathring{N} = (n^{\mathrm{r}}\ n^{\mathrm{r}}\ \bar{\kappa})^{-1} N$. As the economy grows large, i.e. as n^{r} grows to infinity, we have the following expression:

$$\check{H}_{jt} = \Bigg[\int\Bigg(\mathbb{E}_{r'}\Bigg[\Bigg(\mathbb{E}_{j'\in J_{r'}}\Bigg[\left(X^{\lambda\theta_{j'}}\tau\overline{G}_{r't}\tilde{G}_{j't}\mathring{G}_{j'}(X)C_{r'}^{\lambda}\check{H}_{j't}^{-\alpha_{r'}\lambda}\right)^{\beta/\rho_{r'}}A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}\Bigg]\Bigg)^{\rho_{r'}}\Bigg]\Bigg)^{-1}$$
$$\times\left(\mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\overline{G}_{rt}\tilde{G}_{j't}\mathring{G}_{j'}(X)C_r^{\lambda}\check{H}_{j't}^{-\alpha_r\lambda}\right)^{\beta/\rho_r}A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right]\right)^{\rho_r-1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau\overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.$$

Next we show that $\check{H}_{jt}$ can indeed be expressed as stated in this Lemma. Let's assume that $\check{H}_{jt} = \check{H}_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} \cdot \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \cdot \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}}$ and show that it solves the problem. Note that

$$
\begin{aligned}
&\mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\overline{G}_{rt}\tilde{G}_{j't}\mathring{G}_{j'}(X)C_r^{\lambda}\check{H}_{j't}^{-\alpha_r\lambda}\right)^{\beta/\rho_r}A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right]\\
&\quad=\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}\overline{G}_{rt}^{(1-\frac{\alpha\lambda\beta}{1+\alpha_r\lambda\beta})\beta/\rho_r}\mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_{j'}(X)C_r^{\lambda}\check{H}_{j'}^{-\alpha_r\lambda}\right)^{\beta/\rho_r}\tilde{G}_{j't}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right]\\
&\quad=\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}\overline{G}_{rt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta}}\check{I}_{r0}(X)^{\lambda\beta/\rho_r}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\check{H}_{jt}=&\left[\int\left(\mathbb{E}_{r'}\left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}}\overline{G}_{r't}^{\frac{\beta}{1+\alpha_r\lambda\beta}}\check{I}_{r'0}(X)^{\lambda\beta}\right]\right)^{-1}\times\left(\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}\overline{G}_{rt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta}}\check{I}_{r0}(X)^{\lambda\beta/\rho_r}\right)^{\rho_r-1}\right.\\
&\quad\left.\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau\overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}\\
=&\left[\int X^{\theta_j}\left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)}\right)^{\lambda\beta}\left(\frac{1}{\check{I}_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}\\
&\quad\times\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}\frac{\rho_r-1}{1+\alpha_r\lambda\beta/\rho_r}}\times\overline{G}_{rt}^{\frac{\beta/\rho_r+\frac{(\rho_r-1)\beta/\rho_r}{1+\alpha_r\lambda\beta}}{1+\alpha_r\lambda\beta/\rho_r}}\times\tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\\
=&\,\check{H}_j\cdot\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}\cdot\overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}}\cdot\tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}},
\end{aligned}
$$

where we used that $\check{H}_j$ solves

$$
\check{H}_j=\left[\int X^{\theta_j}\left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)}\right)^{\lambda\beta}\left(\frac{1}{\check{I}_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.
$$

□

COROLLARY 4: *Allowing for time-varying amenities $G_{jt}(X)=\overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X)$ we get the following wage equation:*

$$
w_{jt}(x)=c_r+\theta_j x-\alpha_r\check{h}_j+\frac{\tilde{a}_{jt}-\alpha_r\beta/\rho_r\cdot\tilde{g}_{jt}}{1+\alpha_r\lambda\beta/\rho_r}+\frac{\overline{a}_{rt}-\alpha_r\beta\cdot\overline{g}_{rt}}{1+\alpha_r\lambda\beta}.
$$

6. *An extension with capital and monopolistic competition in the product market*

We develop here a simple extension of the model with capital and monopolistic competition in the product market. Without loss of generality, we derive the results here in the case of homogeneous labor.

Consider a firm with production function $Q = AK^{\rho}L^{1-\alpha}$, access to a local monopolistic market with revenue curve $Y = Q^{1-\epsilon}$, hiring labor from a local labor supply curve $L(W) = W^{\beta}$ and renting capital at price r. Profit is given by

$$Q^{1-\epsilon} - LW - rK.$$

We first note that we can replace Q with the production function and get

$$\left(AK^{\rho}L^{1-\alpha}\right)^{1-\epsilon} - LW - rK.$$

Now we will show that considering perfect or monopolistic competition in the product market gives rise to the same revenue function. We will focus directly on the value added function parameterized as

$$Y = AK^{\tilde{\rho}}L^{1-\tilde{\alpha}},$$

where $\tilde{\rho} \equiv \rho(1-\epsilon)$ and $\tilde{\alpha} \equiv \alpha + \epsilon - \alpha\epsilon$. We then have the following Lagrangian for our problem:

$$AK^{\tilde{\rho}}L^{1-\tilde{\alpha}} - LW - rK - \mu(L - W^{\beta}).$$

We take the first order condition for K and get

$$K = \left(\frac{r}{\tilde{\rho}AL^{1-\tilde{\alpha}}}\right)^{\frac{1}{\tilde{\rho}-1}},$$

which we then replace in

$$\begin{aligned}
AK^{\tilde{\rho}}L^{1-\tilde{\alpha}} - LW - rK &= A\left(\frac{r}{\tilde{\rho}AL^{1-\tilde{\alpha}}}\right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\tilde{\alpha}} - LW - r\left(\frac{r}{\tilde{\rho}AL^{1-\tilde{\alpha}}}\right)^{\frac{1}{\tilde{\rho}-1}} \\
&= (1-\tilde{\rho})A\left(\frac{r}{\tilde{\rho}AL^{1-\tilde{\alpha}}}\right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\tilde{\alpha}} - LW \\
&= (1-\tilde{\rho})A\left(\frac{r}{\tilde{\rho}A}\right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\frac{\tilde{\alpha}+\tilde{\rho}}{1-\tilde{\rho}}} - LW \\
&= \hat{A}L^{1-\hat{\alpha}} - LW,
\end{aligned}$$

which is just a reinterpretation of the original problem with $\hat{A} \equiv (1-\tilde{\rho})A\left(\frac{r}{\tilde{\rho}A}\right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}}$, $\hat{\alpha} \equiv \frac{\tilde{\alpha}+\tilde{\rho}}{1-\tilde{\rho}}$.

B. Details on Data Sources and Sample Selection

All firm level variables are constructed from annual business tax returns over the years 2001-2015: C-Corporations (Form 1120), S-Corporations (Form 1120-

S), and Partnerships (Form 1065). Worker-level variables are constructed from annual tax returns over the years 2001-2015: Direct employees (Form W-2), independent contractors (Form 1099), and household income and taxation (Form 1040).

Variable Definitions:

- **Earnings:** Reported on W-2 box 1 for each Taxpayer Identification Number (TIN). Each TIN is de-identified in our data.
- **Gross Household Income:** We define gross household income as the sum of taxable wages and other income (line 22 on Form 1040) minus unemployment benefits (line 19 on Form 1040) minus taxable Social Security benefits (line 20a on Form 1040) plus tax-exempt interest income (line 8b on Form 1040). We at times also consider this measure when subtracting off Schedule D capital gains (line 13 on Form 1040).
- **Federal Taxes on Household Income:** This is given by the sum of two components. The first component is the sum of FICA Social Security taxes (given by 0.0620 times the minimum of the Social Security taxable earnings threshold, which varies by year, and taxable FICA earnings, which are reported on Box 3 of Form W-2) and FICA Medicare taxes (given by 0.0145 times Medicare earnings, which are reported on Box 5 of Form W-2). The second component is the sum of the amount of taxes owed (the difference between line 63 and line 74 on Form 1040, which is negative to indicate a refund) and the taxes already paid or withheld (the sum of lines 64, 65, 70, and 71 on Form 1040).
- **Net Household Income:** We construct a measure of net household income as Gross Household Income minus Federal Taxes on Household Income plus two types of benefits: unemployment benefits (line 19 of Form 1040) and Social Security benefits (line 20a of Form 1040).
- **Employer:** The Employer Identification Number (EIN) reported on W-2 for a given TIN. Each EIN is de-identified in our data.
- **Wage Bill:** Sum of Earnings for a given EIN plus the sum of 1099-MISC, box 7 nonemployee compensation for a given EIN in year t.
- **Size:** Number of FTE workers matched to an EIN in year t.
- **NAICS Code:** The NAICS code is reported on line 21 on Schedule K of Form 1120 for C-corporations, line 2a Schedule B of Form 1120S for S-corporations, and Box A of form 1065 for partnerships. We consider the first two digits to be the industry. We code invalid industries as missing.
- **Commuting Zone:** This is formed by mapping the ZIP code from the business filing address of the EIN on Form 1120, 1120S, or 1065 to its commuting zone.

- **Value Added:** Line 3 of Form 1120 for C-Corporations, Form 1120S for S-Corporations, and Form 1065 for partnerships. Line 3 is the difference between Revenues, reported on Line 1c, and the Cost of Goods Sold, reported on Line 2. We replace non-positive value added with missing values.
  - For manufacturers (NAICS Codes beginning 31, 32, or 33) and miners (NAICS Codes beginning 212), Line 3 is equal to Value Added minus Production Wages, defined as wage compensation for workers directly involved in the production process, per Schedule A, Line 3 instructions. If we had access to data from Form 1125-A, Line 3, we could directly add back in these production wages to recover value added. Without 1125-A, Line 3, we construct a measure of Production Wages as the difference between the Wage Bill and the Firm-reported Taxable Labor Compensation, defined below, as these differ conceptually only due to the inclusion of production wages in the Wage Bill.
- **Value Added Net of Depreciation:** Value Added minus Depreciation, where Depreciation is reported on Line 20 on Form 1120 for C-corporations, Line 14 on Form 1120S for S-corporations, and Line 16c on Form 1065 for partnerships.
- **EBITD:** We follow Kline et al. (2019) in defining Earnings Before Interest, Taxes, and Depreciation (EBITD) as the difference between total income and total deductions other than interest and depreciation. Total income is reported on Line 11 on Form 1120 for C-corporations, Line 1c on Form 1120S for S-corporations, and Line 1c on Form 1065 for Partnerships. Total deductions other than interest and depreciation are computed as Line 27 minus Lines 18 and 20 on Form 1120 for C-corporations, Line 20 minus Lines 13 and 14 on Firm 1120S for S-corporations, and Line 21 minus Lines 15 and 16c on Form 1065 for partnerships.
- **Operating Profits:** We follow Kline et al. (2019), who use a similar approach to Yagan (2015), in defining Operating Profits as the sum of Lines 1c, 18, and 20, minus the sum of Lines 2 and 27 on Form 1120 for C-corporations,, the sum of Lines 1c, 13, and 15, minus the sum of Lines 2 and 20 on Form 1120S for S-corporations, and the sum of Lines 1c, 16, and 16c, minus the sum of Lines 2 and 21 on Form 1065 for partnerships.
- **Firm-reported Taxable Labor Compensation:** This is the sum of compensation of officers and salaries and wages, reported on Lines 12 and 13 on Form 1120 for C-corporations, Lines 7 and 8 on Form 1120S for S-corporations, and Lines 9 and 10 on Form 1065 for Partnerships.
- **Firm-reported Non-taxable Labor Compensation:** This is the sum of employer pension and employee benefit program contributions, reported on Lines 17 and 18 on Form 1120 for C-corporations, Lines 17 and 18

on form 1120S for S-corporations, and Lines 18 and 19 on Form 1065 for Partnerships.

- **Multinational Firm:** We define an EIN as a multinational in year t if it reports a non-zero foreign tax credit on Schedule J, Part I, Line 5a of Form 1120 or Form 1118, Schedule B, Part III, Line 6 of Form 1118 for a C-corporation in year t, or if it reports a positive Total Foreign Taxes Amount on Schedule K, Line 16l of of Form 1065 for a partnership in year t.
- **Tenure:** For a given TIN, we define tenure at the EIN as the number of consecutive prior years in which the EIN was the highest-paying.
- **Age and Sex:** Age at t is the difference between t and birth year reported on Data Master-1 (DM-1) from the Social Security Administration, and sex is the gender reported on DM-1 (see for further details on the DM-1 link).

C. Details on Identification, Estimation, and Robustness

1. *Moment condition for internal panel instruments*

In this appendix, we prove that equation (12) holds. Using equations (4), (5), and (10), we can write for the stayers (S_i=1) that

$$\begin{aligned}\tilde{y}_{it+\tau} - \tilde{y}_{j,t-\tau'} =& \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'} + \nu_{j,t+\tau} - \nu_{j,t-\tau'}\\ \tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} =& v_{it+\tau} - v_{i,t-\tau'} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'}\end{aligned}$$

Combining these equations, it follows that

$$\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right) = -\frac{1}{1+\lambda\beta/\rho_r}\left(\nu_{j,t+\tau} - \nu_{j,t-\tau'}\right) + v_{it+\tau} - v_{i,t-\tau'}$$

Furthermore, the short-difference in log value added can be written

$$\Delta\tilde{y}_{j(i),t} = \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{u}_{j(i),t} + \nu_{j,t} - \nu_{j,t-1}$$

Combining these expressions and taking the expectation,

$$\begin{aligned}&\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \gamma\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]\\ &= \mathbb{E}\left[\left(\frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{u}_{j(i),t} + \nu_{j,t} - \nu_{j,t-1}\right)\left(-\frac{1}{1+\lambda\beta/\rho_r}\left(\nu_{j,t+\tau} - \nu_{j,t-\tau'}\right) + v_{it+\tau} - v_{i,t-\tau'}\right)|S_i{=}1\right]\end{aligned}$$

Given Assumption 2 that $\mathbb{E}\left[\nu_{jt}\nu_{j't}|\Omega_T\right]=0$ whenever $|t-t'|\geq 2$, it follows that whenever $\tau \geq 2$ and $\tau' \geq 3$, all cross-products between ν_{jt} terms will be mean zero. Furthermore, $\mathbb{E}\left[\nu_{jt}|\Omega_T\right]=0$ ensures that cross-product terms between $\tilde{u}_{jt}$ and ν_{jt} are also mean zero. Finally the assumption that the measurement error on wages is independent of all firm level variables, Assumption 3, implies that all terms involving v_{it} are also mean zero. Thus, provided that $\tau \geq 2$ or $\tau' \geq 3$,

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau'}-\frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]=0.$$

As a result, $\frac{1}{1+\lambda\beta/\rho_r}\equiv\gamma_r$ is identified as long as,

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau'}\right)|S_i{=}1\right]>0,$$

which is guaranteed by Assumption 1.

A similar argument can be used to establish that equation (13) holds. Briefly, among stayers, market level changes in log wages and log value added are given by

$$\begin{aligned}\bar{w}_{it+\tau}-\bar{w}_{it-\tau}=&\frac{1}{1+\lambda\alpha_{r(j(i))}\beta}\sum_{d=t-\tau'+1}^{t+\tau}\overline{u}_{r(j(i)),d}\\ \bar{y}_{jt+\tau}-\bar{y}_{jt-\tau'}=&\frac{1+\lambda\beta}{1+\lambda\alpha_{r(j)}\beta}\sum_{d=t-\tau'+1}^{t+\tau}\overline{u}_{r(j),d}\end{aligned}$$

which cancel out differences to imply the moment condition

$$\mathbb{E}\left[\Delta\bar{y}_{j(i),t}\left(\bar{w}_{it+\tau}-\bar{w}_{it-\tau'}-\frac{1}{1+\lambda\beta}\left(\bar{y}_{j(i),t+\tau}-\bar{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]=0.$$

Similarly, the rank condition is guaranteed by Assumption 1, so $\frac{1}{1+\lambda\beta}\equiv\Upsilon$ is identified.

2. *Estimating the rest of the process parameters*

In this appendix, we describe the estimation procedure for recovering the joint process for log earnings and value added. We rely on the assumed structure that each evolves according to a unit root process plus a moving average process, where both the transitory and permanent shocks to value added pass-through to log earnings. We estimate the pass-through process in two steps. First, we estimate the parameters for the value added process. Second, we jointly estimate the pass-through rates at the firm and market level and the parameters of the wage process.

To estimate the value added process, we consider the variance-covariance matrix of one-year differences over time in a stacked panel of 8-year stayer spells. We index the 8-year spells by event times $e = 1, ..., 8$. The variance-covariance matrix uses the growth at event times $e = 3, ..., 7$.[1] For example, the growth in log value added at event time e means the log value added at e minus log value added at $e - 1$. We do not use data from the first ($e = 1$) or last ($e = 8$) year of the spell. We do this because first and last event years can be partial employment spells due to beginning or ending the job spell mid-year. Thus, focusing on the intermediate event years alleviates the issue that we do not observe the exact date at which a job spell begins or ends in our data.

Using our data, we estimate the 5×5 variance-covariance matrix of one-year changes in log value added, denoted M_y, where the (p, q) element is $M_y(p, q) = Cov(\Delta y_{ip}, \Delta y_{iq})$. We construct the analogous population variance-covariance matrix implied by the model as a function of only the parameters $\{\delta^y, \sigma_u, \sigma_\xi\}$; we denote the model-implied variance-covariance matrix by $M_y^*(\delta^y, \sigma_u, \sigma_\xi)$. Given these moments, our GMM estimator solves the minimum distance problem defined by

$$\min_{\delta^y, \sigma_u, \sigma_\xi} \sum_{p=3}^{7} \sum_{q=3}^{7} W_y(p, q) \left(M_y^*(p, q; \delta^y, \sigma_u, \sigma_\xi) - M_y(p, q) \right)^2$$

where we use diagonal weighting, i.e., $W_y(p, q) = Cov(\Delta y_{ip}, \Delta y_{iq})^2 + Var(\Delta y_{ip}) Var(\Delta y_{iq})$.

Next, we construct two matrices each of size 5×5. The first, M_w, is the variance-covariance matrix for one-year changes in log wages; a typical element is $M_w(p, q) = Cov(\Delta w_{ip}, \Delta w_{iq})$. The second, M_{wy}, is the variance-covariance matrix for one-year changes in log wages and log value added; a typical element is $M_{wy}(p, q) = Cov(\Delta w_{ip}, \Delta y_{iq})$. The corresponding model-implied population variance-covariance matrices are $M_w^*(\delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta)$ and $M_{wy}^*(\delta^w, \sigma_\mu, \sigma_\nu)$, respectively. These matrices also depend on $(\delta^y, \sigma_u, \sigma_\xi)$, which were estimated in the first step, so we substitute in to $M_w^*(\delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta)$ and $M_{wy}^*(\delta^w, \sigma_\mu, \sigma_\nu)$ the estimated values of $(\delta^y, \sigma_u, \sigma_\xi)$.Then, our GMM estimator in the second step solves the minimum distance problem defined by

$$\min_{p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta} \sum_{p=3}^{7} \sum_{q=3}^{7} W_w(p, q) \left(M_w^*(p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta) - M_w(p, q) \right)^2 +$$
$$W_{wy}(p, q) \left(M_{wy}^*(p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta) - M_{wy}(p, q) \right)^2$$

where we again use diagonal weighting, i.e., $W_w(p, q) = Cov(\Delta w_{ip}, \Delta w_{iq})^2 + Var(\Delta w_{ip}) Var(\Delta w_{iq})$ and $W_{wy}(p, q) = Cov(\Delta w_{ip}, \Delta y_{iq})^2 + Var(\Delta w_{ip}) Var(\Delta y_{iq})$.

In practice, the GMM minimum distance problems in the first and second steps are polynomials in the parameters of interest. We solve the minimization problems

[1] In the case of MA(1), one can also use $t = 2$, however we wanted to test for MA(2) as a robustness.

using global polynomial optimization following Lasserre (2001). This allows us to formally certify the global optimality of the solution.

For inference, we use a joint bootstrap of M_y, M_w, M_{yw}. We conduct inference using a block bootstrap that resamples markets, where a market is definedas the combination ofa commuting zone an an industry. In practice, thereare about 2000 blocks. The GMM estimates and bootstrap standard errors are displayed in Online Appendix Table A.3.

3. Pass-through estimation based on external instruments

Identification details

Implicitly conditioning on firms in region r ($j(i,t) = j \in J_r$), we prove that this claim from the main text holds:

$$\mathbb{E}\left[\Delta\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e}-\tilde{w}_{it-e'} - \gamma_r\left(\tilde{y}_{jt+e}-\tilde{y}_{jt-e'}\right)\right)|S_i{=}1\right] = 0$$

From equations (4), (5), and the expression for h_{jt} in Appendix A.5, we have that

$$\tilde{w}_{it+e}-\tilde{w}_{it-e'} = -\alpha_r\left(h_{j(i)t+e} - h_{j(i)t-e}\right) + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right) + \left(v_{it+e} - v_{it-e}\right)$$

$$\tilde{y}_{j(i)t+e}-\tilde{y}_{j(i)t-e'} = (1-\alpha_r)\left(h_{j(i)t+e} - h_{j(i)t-e}\right) + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right) + \left(\tilde{\nu}_{j(i)t+e} - \tilde{\nu}_{j(i)t-e}\right)$$

From assumption 4,

$$\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e}-\tilde{a}_{j(i)t-e}\right)|S_i{=}1\right] \neq 0$$

$$\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e}-h_{j(i)t-e'}\right)|S_i{=}1\right] = 0$$

It follows that

$$\begin{aligned}
\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e}-\tilde{w}_{it-e'}\right)|S_i{=}1\right] &= -\alpha_r \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e}-h_{j(i)t-e}\right)|S_i{=}1\right]}_{=0} \\
&+ \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e}-\tilde{a}_{j(i)t-e}\right)|S_i{=}1\right]}_{\neq 0} \\
&+ \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(v_{it+e}-v_{it-e}\right)|S_i{=}1\right]}_{=0} \\
\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{y}_{j(i)t+e}-\tilde{y}_{j(i)t-e'}\right)|S_i{=}1\right] &= (1-\alpha_r)\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e}-h_{j(i)t-e}\right)|S_i{=}1\right]}_{=0} \\
&+ \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e}-\tilde{a}_{j(i)t-e}\right)|S_i{=}1\right]}_{\neq 0} \\
&+ \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\nu_{j(i)t+e}-\nu_{j(i)t-e}\right)|S_i{=}1\right]}_{=0}
\end{aligned}$$

where we imposed the restrictions (2 part i) and 3) to eliminate the terms involving the measurement errors v_{it} and ν_{jt}. Thus, we can write

$$\frac{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e}-\tilde{w}_{it-e'}\right)|S_i{=}1\right]}{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{y}_{j(i)t+e}-\tilde{y}_{j(i)t-e'}\right)|S_i{=}1\right]} = \frac{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} = \frac{1}{1+\lambda\beta/\rho_r} \equiv \gamma_r$$

which can be rearranged as the claim above. The same reasoning demonstrates the claim that Υ can be identified using $\overline{\Lambda}_{rt}$.

Procurement auction shocks at firm-level

Our goal is to recover the pass-through regression at the firm-level. Following the research design of Kroft et al. (2021), consider the cohort of firms that received a procurement contract in year t ($D_{jt}=1$) and the set of comparison firms that bid for a procurement in year t but lost ($D_{jt}=0$). Let e denote an event time relative to t and $\bar{e}$ denote the omitted event time. For each event time $e=-4,...,4$, the DiD regression is implemented as

$$w_{jt+e} = \underbrace{\sum_{e'\neq\bar{e}} 1\left\{e'=e\right\}\mu_{te'}}_{\textit{event time fixed effect}} + \underbrace{\sum_{j'} 1\left\{j'=j\right\}\psi_{j't}}_{\textit{firm fixed effect}} + \underbrace{\sum_{e'\neq\bar{e}} 1\left\{e'=e\right\}D_{jt}\vartheta_{te'}}_{\textit{treatment status by event time}} + \underbrace{\nu_{jte}}_{\textit{residual}}$$

We report the average across t of the estimated ϑ_{te} parameters, which can be interpreted as the reduced form effect on log earnings of receiving an exogenous demand shock, that is, $\vartheta_{te} = \mathbb{E}\left[w_{jt+e} - w_{t-\bar{e}} | D_{jt} = 1\right] - \mathbb{E}\left[w_{jt+e} - w_{t-\bar{e}} | D_{jt} = 0\right]$. We estimate ϑ_{te} for all t and e and then average across t, using the delta method to compute standard errors (which are clustered at the firm level j to account for serial correlation). By doing so, we avoid the problem that cohorts can be negatively weighted in pooled cohort DiD estimators. The analogous regression in which y_{jt+e} is the outcome recovers the first stage effect on log value added, $\mathbb{E}\left[y_{jt+e} - y_{t-\bar{e}} | D_{jt} = 1\right] - \mathbb{E}\left[y_{jt+e} - y_{t-\bar{e}} | D_{jt} = 0\right]$. The ratio of the reduced form effect and the first stage effect yields the second stage effect, which is the pass-through coefficient γ. In the first panel in Appendix Table A.4, we apply this research design to the sample of 8,667 unique firms that bid in the sample of procurement auctions administered by the departments of transportation in 28 states during 2001-2015. We refer to Kroft et al. (2021) for details on how the procurement auction data were collected and linked to IRS tax records as well as institutional details and descriptive statistics. We find a statistically significant first stage coefficient of 0.143, indicating that winners of procurement auctions experience about 14 percent more growth in value added than losers of procurement contracts. We find a statistically significant reduced form coefficient of 0.020, indicating that workers employed by firms that win procurement auctions experience about 2 percent more growth in earnings than workers employed by losers of procurement contracts. The ratio of the reduced form and first stage effects yields a statistically significant firm-level pass-through coefficient γ of 0.142.

Shift share industry value added shock

In order to provide IV estimates of the market level pass-through and labor supply elasticity, we follow Bartik (1991) and Blanchard and Katz (1992) in constructing a shift-share instrument. Let cz denote a commuting zone and ind denote a 2-digit NAICS industry, and recall that a market is defined by the pair (cz, ind) in our main specification. Let $\overline{Y}_{cz,ind,t}$ and $\overline{W}_{cz,ind,t}$ denote the total value added and total earnings per worker of stayers in the (cz, ind) at time t, and $\overline{Y}_{ind,t} \equiv \sum_{cz} \overline{Y}_{cz,ind,t}$ denote aggregate industry value added. Let $\overline{Y}_{cz,t} \equiv \sum_{ind} \overline{Y}_{cz,ind,t}$ and $\overline{W}_{cz,t} \equiv \sum_{ind} \overline{W}_{cz,ind,t}$ denote aggregate commuting zone value added and earnings per stayer, respectively.

Then, the shift-share value added shock to the commuting zone is constructed as $\sum_{ind} S_{cz,ind,t_0} \zeta_{ind,t}$, where $S_{cz,ind,t} \equiv \overline{Y}_{cz,ind,t} / \overline{Y}_{cz,t}$ is the exposure of the cz to a particular ind (the "share" component), $\zeta_{ind,t} \equiv \log \overline{Y}_{ind,t} - \log \overline{Y}_{ind,t-\tau}$ is the log change in industry value added (the "shift" component), and we measure the share component at the earliest period in the sample (i.e., t_0 =2001). To estimate the market level pass-through, we regress the log change in earnings per stayer $\log \overline{W}_{cz,t} - \log \overline{W}_{cz,t-\tau}$ in the commuting zone on the log change in total value added in the commuting zone $\log \overline{Y}_{cz,t} - \log \overline{Y}_{cz,t-\tau}$, instrumented by the shift-share value added shock.

In order to draw statistical inference, we cluster standard errors at the industry-level using the approach of Borusyak, Hull and Jaravel (Forthcoming). To do so, we transform the outcome variable $\log \overline{W}_{cz,t} - \log \overline{W}_{cz,t-\tau}$ and the endogenous regressor $\log \overline{Y}_{cz,t} - \log \overline{Y}_{cz,t-\tau}$ into industry-level variables using the equivalence result in Proposition 1 of Borusyak, Hull and Jaravel (Forthcoming). Then, we regress the industry-level transformed outcome variable on the industry-level transformed endogenous regressor, instrumented by the industry-level shock $\zeta_{ind,t}$, and calculate heteroskedasticity-robust standard errors. In the second panel in Appendix Table A.4, we apply this research design to the sample of 667 unique commuting zones during 2001-2015. The ratio of the reduced form and first stage effects yields a statistically significant market-level pass-through coefficient Υ of 0.189. The first stage F-statistic using only industry-level variation is about 11.

4. Interacted fixed effect equation, firm specific TFP a_{jt} and amenities h_j

Identification details

We consider the equation in the text,

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{rt} - \bar{y}_{r1}\right) - \frac{\rho_r}{\rho_r + \lambda\beta}(\tilde{y}_{jt} - \tilde{y}_{j1}) \middle| \begin{array}{c} j(i,t)=j \\ j \in J_r \end{array} \right].$$

We assume that the initial conditions for the permanent productivity shocks at the firm and market level satisfy $\tilde{a}_{j1} = \tilde{p}_j$ and $\overline{a}_{r(j)1} = \overline{p}_r$. Then, we can write

$$\begin{aligned} w_{it} &= \theta_j x_i + c_r - \alpha_r h_{j(i,t)} + \frac{1}{1+\lambda\alpha_r\beta/\rho_r}\tilde{a}_{j(i,t)t} + \frac{1}{1+\lambda\alpha_r\beta}\overline{a}_{r(j(i,t))t} + v_{it} \\ \tilde{y}^*_{j,t} - \tilde{y}^*_{j1} &= \frac{1+\lambda\beta/\rho_r}{1+\lambda\alpha_r\beta/\rho_r}\left(\tilde{a}_{jt} - \tilde{p}_j\right) \\ \bar{y}^*_{rt} - \bar{y}^*_{r1} &= \frac{1+\lambda\beta}{1+\lambda\alpha_r\beta}\left(\overline{a}_{rt} - \overline{p}_r\right) \end{aligned}$$

where $\tilde{y}^*_{j,t}$ and $\bar{y}^*_{rt}$ denote $\tilde{y}_{j,t}$ and $\bar{y}_{rt}$ net of measurement error. Given that the measurement error in y_{jt}, ν_{jt}, is mean zero and the same applies to the measurement error in w_{it}, v_{it}, even conditional on mobility (as given by assumptions 2 and 3), we have that

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{rt} - \bar{y}_{r1}\right) - \frac{\rho_r}{\rho_r + \lambda\beta}(\tilde{y}_{jt} - \tilde{y}_{j1}) \middle| \begin{array}{c} j(i,t)=j \\ j \in J_r \end{array} \right] = \theta_j x_i + \psi_j,$$

where we define

$$\psi_j \equiv c_r - \alpha_r h_j + \frac{1}{1+\lambda\beta}\overline{p}_r + \frac{\rho_r}{\rho_r+\lambda\beta}\tilde{p}_j.$$

Next, we can identify θ_j from data on the changes in earnings associated with these moves:

$$(5) \qquad \frac{\mathbb{E}[w^a_{it+1}|j(i,t)=j', j(i,t+1)=j] - \mathbb{E}[w^a_{it}|j(i,t)=j, j(i,t+1)=j']}{\mathbb{E}[w^a_{it}|j(i,t)=j', j(i,t+1)=j] - \mathbb{E}[w^a_{it+1}|j(i,t)=j, j(i,t+1)=j']} = \frac{\theta_j}{\theta_{j'}}$$

as long as the denominator is non zero, which is ensured by the following assumption:

$$\mathbb{E}\left[x_i|j(i,t)=j, j(i,t+1)=j'\right] \neq \mathbb{E}\left[x_i|j(i,t)=j', j(i,t+1)=j\right].$$

Individual types x_i are then also idenfitied from Assumption 3 since

$$x_i = \mathbb{E}\left[\frac{w^a_{it} - \psi_{j(i,t)}}{\theta_{j(i,t)}}|i\right].$$

Given x_i, we can construct the firm's log efficiency units of labor as

$$l_{jt} = \log \int X^{\theta_j} D_{jt}(X)\mathrm{d}X.$$

Since the production function paramters $\alpha_{r(j)}$ is already known, we get the following expression for a_{jt}:

$$\mathbb{E}\left[y_{jt} - \alpha_{r(j)} l_{jt}|j\right] = a_{jt}.$$

We can use this to construct $\overline{a}_{rt} = \mathbb{E}[a_{jt}|j \in J_r]$ and $\tilde{a}_{jt} = a_{jt} - \overline{a}_{rt}$. This then identifies the permanent components $\tilde{p}_j$ and $\tilde{p}_r$ as well as the inovation variances $\sigma^2_{\tilde{u}}$ and $\sigma^2_{\overline{u}}$. The final step is to rearrange the expression for ψ_j to back out h_j.

Estimation details

Equation (14) and (20) make clear that (ψ_j, θ_j) can be identified from comparing the gains from moving from a low to a high type of firm for workers of different quality. In practice, we simultaneously recover (ψ_j, θ_j) from the following moment condition:

$$(6) \qquad \mathbb{E}\left[\left(\frac{w^a_{it+1}}{\theta_{j'}} - \frac{\psi_{j'}}{\theta_{j'}}\right) - \left(\frac{w^a_{it}}{\theta_j} - \frac{\psi_j}{\theta_j}\right)\Big|j(i,t)=j, j(i,t+1)=j'\right] = 0.$$

This moment condition provides an instrumental variables representation where the interactions between indicators for firm before the move and firm after the move can be interpreted as the instruments and the parameters are $(\frac{1}{\theta_1}, ..., \frac{1}{\theta_K}, \frac{\psi_1}{\theta_1}, ..., \frac{\psi_K}{\theta_K})$.

In the general case in which the number of firm types is unrestricted, $\left(\hat{\theta}_j, \hat{\psi}_j\right)$ would suffer from incidental parameter bias, even under the assumption that $\theta_j = 1$ (see the discussion by Bonhomme et al. 2020). As discussed in the text and further explored in our Online Supplement, we alleviate this concern using the grouped fixed effect estimation with 10 firm types proposed by Bonhomme, Lamadon and Manresa (2019). With 10 firm types, equation (21) provides 100 moments and 20 unknown parameters. As a result, this can be interpreted as an over-identified model. Following Bonhomme, Lamadon and Manresa (2019), estimation is implemented using LIML on these moment conditions where the θ_j are concentrated on the post-move time period (in theory they can be estimated without imposing stationarity). To check the relevance of these instruments, we compute the F-statistic corresponding to the first-stage regression, which is 9288 with an R-squared of about 0.30.

Regaring the estimation of x_i, we use a sample analog and compute $\hat{x}_i = \frac{1}{T}\sum_t \frac{w^a_{it} - \psi_{k(j(i,t))}}{\theta_{k(j(i,t))}}$. Given (θ_j, ψ_j), this is an unbiased estimate of x_i under Assumption 3 and the structure of the wage equation. Yet, the plug-in estimator for the variance of x_i can be biased and inconsistent even asymptotically as the number of workers within each firm type grows large. In our Online Supplement, we consider the additional assumption that the measurement error in log earning is the sum of unit root and an MA(0) term. This allows us to compute the implied bias in the plug in estimator of the variance of x_i in finite T. Under this assumption, we find that the bias in the estimated variance of x_i is very small in our context.

5. *Identification and estimation of $G_j(X)$*

LEMMA 9: *We show that for all $t, j \in J_r, r, X$ we have:*

$$\tau \exp(\lambda \psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \mid X]\right)^{1/\beta} \left(Pr[j(i,t) = j \mid X, j(i,t) \in J_r]\right)^{\rho_r/\beta}.$$

PROOF:We have that:

$$Pr[j(i,t) = j \mid X, j(i,t) \in J_r] = \frac{\left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}}$$

$$Pr[j(i,t) \in J_r \mid X] = \frac{\left(\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}}$$

We can fix a given t and write $G_j(X) = \overline{G}_r(X)\tilde{G}_j(X)$, imposing the normalization that

$$\sum_{j' \in J_r} \left(\tau^{1/\lambda} \tilde{G}_{j'}(X)^{\frac{1}{\lambda}} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r} = 1$$

$$\sum_r \overline{G}_r(X)^\beta = 1$$

Substituting, we have

$$Pr[j(i,t) = j \mid X, j(i,t) \in J_r] = \left(\tau^{1/\lambda} \tilde{G}_j(X)^{\frac{1}{\lambda}} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}$$

$$Pr[j(i,t) \in J_r \mid X] = \left(\overline{G}_r(X)\right)^\beta$$

Thus,

$$\tau \exp(\lambda\psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \mid X]\right)^{1/\beta} \left(Pr[j(i,t) = j \mid X, j(i,t) \in J_r]\right)^{\rho_r/\beta}$$

Since this result does not depend on the normalization, it is true for all t.

□

Next, we explain the estimation procedure that relies on the expression that we just derived. For estimation, we use grouped structure both at the firm and at the market level. We group firms using the classification described in the text based on the firm-specific empirical distribution of earnings; we denote the firm groups by $k(j)$. We follow a similar approach at the market level and group markets based on the market level empirical distribution of earnings; we denote the market groups by $m(r)$. At this point we think of a firm class $k(j)$ as being within market type m, so when using the classification from the main text, we interact the firm group k with the market group m.

Using these two classifications, we rely on the fact that worker composition can be estimated at the group level instead of trying to estimate a distribution for each individual firm and market. Indeed, in the model we have that:

$$Pr[X|j] = Pr[X|k(j)]$$

$$Pr[X|r] = Pr[X|m(r)].$$

Similarly to the Lemma above, we can define $G_j(X) = \overline{G}_r \tilde{G}_j G_{k(j)}(X)$. Following the lemma we impose the following constraints on $\overline{G}_r$ and $\tilde{G}_j$:

$$\sum_{j' \in J_r} \left(\tau^{1/\lambda} \left(\tilde{G}_{j'} G_{k(j')}(X)\right)^{\frac{1}{\lambda}} \exp(\lambda\psi_{j't}) X^{\lambda\theta_{j'}}\right)^{\lambda\beta/\rho_r} = 1$$

$$\sum_r \overline{G}_r^\beta = 1$$

We then directly apply the formula for $G_j(X)$ at the firm group level $k(j)$ within market $m(r(j))$):

$$G_k(X) = X^{-\lambda\theta_k}\left(\frac{Pr[X|m]}{Pr[X]}\right)^{1/\beta}\left(\frac{Pr[X|k]}{Pr[X|m]}\right)^{\rho_r/\beta}.$$

Next we recover the j-specific component by matching the size of each firm within its market:

$$Pr[j(i,t)=j\,|\,j(i,t)\in J_r] = \tilde{G}_j^{\beta/\rho_r}\int\frac{\left(\tau G_{k(j)}(X)\exp(\lambda\psi_{jt})X^{\lambda\theta_j}\right)^{\beta/\rho_r}}{\sum_{j'\in J_r}\left(\tau\tilde{G}_{j'}G_{k(j')}(X)\exp(\lambda\psi_{j't})X^{\lambda\theta_{j'}}\right)^{\beta/\rho_r}}Pr[X|m(r)]\mathrm{d}X$$

Similarly, we recover the market level constant by matching the market level size:

$$Pr[j(i,t)\in J_r\,|\,X] = \overline{G}_r^{\beta}\int\frac{\left(\sum_{j'\in J_r}\left(\tau\tilde{G}_j G_{k(j)}(X)\exp(\lambda\psi_{j't})X^{\lambda\theta_{j'}}\right)^{\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'}\left(\sum_{j'\in J_{r'}}\left(\tau\overline{G}_{r'}\tilde{G}_{j'}G_{k(j')}(X)\exp(\lambda\psi_{j't})X^{\lambda\theta_{j'}}\right)^{\beta/\rho_{r'}}\right)^{\rho_{r'}}}NM(X)\mathrm{d}X$$

D. Additional Robustness Checks

1. Pass-through estimation

The main results are displayed in Online Appendix Table A.3. Additional heterogeneity and robustness analyses are presented in Online Appendix Figure A.1.

We now provide evidence that the main results are not sensitive to alternative specifications. First, we allow for greater persistence in the transitory shock process by considering a MA(2) specification. This is accounted for by choosing $e = 3, e' = 4$ in the empirical counterparts to equations (12)-(13). Results are provided in the fourth column of Panel B in Online Appendix Table A.3. Under an MA(2) specification of the transitory shock process, we estimate that the average firm level pass-through rate γ_r is 0.13 and the market level pass-through rate Υ is 0.18, which are the same as our main findings from the MA(1) specification.

Second, our specification of the earnings process allows permanent shocks to value added to be transmitted to workers' earnings, whereas transitory firm shocks are not. As a specification check, we allow transitory innovations to value added to transmit to workers' earnings. Results are provided in the fourth column of Panel A in Online Appendix Table A.3. We find little if any pass-through of transitory shocks. As a result, transitory shocks explain as little as 0.1 percent of the variation in log earnings. This finding is consistent with previous work (see, e.g., Guiso, Pistaferri and Schivardi 2005; Friedrich et al. 2019). A possible interpretation of this finding is that transitory changes in value added reflect measurement error that do not give rise to economic responses. In the remainder of the paper, we will treat the transitory changes in value added as measurement error and focus on the pass-through of the permanent shocks.

Third, to compare with existing work, we also consider estimating the restricted specification that imposes $\gamma_r = \Upsilon, \forall r$. This is equivalent to imposing $\rho_r = 1, \forall r$, so that idiosyncratic worker preferences over firms are uncorrelated within markets. These results are reported in the first two columns of Panel A in Online Appendix Table A.3. The estimated pass-through rate is then 0.14, which is between our estimates of 0.13 at the firm level and 0.18 at the market level.

Fourth, in Online Appendix Figure A.1, we explore robustness of the pass-through estimates across subsamples of workers. Conditional on a full set of year times market fixed effects, we find in subfigure (a) that the pass-through rates do not vary that much by the worker's age, previous wage, or gender. Moreover, the pass-through rates do not change materially if we restrict the sample to new workers who were first hired at the firm in the beginning of the eight year employment spell versus those that have stayed in the firm for a longer time.

Fifth, in subfigure (b) of in Online Appendix Figure A.1, we present results from several specification checks on firms. Following Guiso, Pistaferri and Schivardi (2005), our main measure of firm performance is value added. They offer two reasons for using value added as a measure of firm performance: value added is the variable that is directly subject to stochastic fluctuations, and firms have discretionary power over the reporting of profits in balance sheets, which makes profits a less reliable objective to assess. Nevertheless, it is reassuring to find that the estimates of the pass-through rates are broadly similar if we measure firm performance by operating profits, earnings before interest, tax and depreciation (EBITD), or value added net of reported depreciation of capital. We also show that the estimated pass-through rate is in the same range as our baseline result if we exclude multinational corporations (for which it can be difficult to accurately measure value added) or exclude the largest firms (that are more likely to have multiple plants, which may not necessarily have the same wage setting).

2. Firm and worker effect estimation

In Online Appendix Table A.6, we provide a number of specification checks. First, we consider estimating the model when ignoring firm-worker interactions by imposing $\theta_j = \bar{\theta}$. The results are presented in the second column of Table A.6. When interactions are ignored, the share of earnings variation explained by worker quality increases by about two percentage points while that explained by firm effects decreases from 4.3 percent to 3.0 percent. Sorting and time-varying effects are little changed. We conclude that the estimated variance of firm effects is downward-biased when ignoring firm-worker production complementarities.

Second, we consider estimating the model when ignoring time-varying effects by imposing $\gamma_r = \Gamma = 0$. The results are presented in the third column of Table A.6. When time-varying effects are ignored, the share of earnings variation explained by worker quality decreases by about one percentage point while that explained by interactions increases by about half a percentage point. The variance of firm effects and sorting are little changed. We conclude that there is little bias in the other terms in the variance decomposition when ignoring production complementarities.

Third, we consider estimating the model when ignoring both firm-worker interactions and time-varying effects by imposing $\theta_j = \bar{\theta}$ and $\gamma_r = \Gamma = 0$. The results are presented in the fourth column of Table A.6. The estimates for worker quality, firm effects, and sorting are similar to the results when only ignoring firm-worker interactions. Note that specification is the same as the model of Abowd, Kramarz and Margolis (1999) that has been estimated in a recent literature except that we use a bias-corrected estimate, so we can compare this specification directly to other papers to learn about limited mobility bias. An extensive discussion of limited mobility bias and comparison to the literature is available in our Online Supplement.

In our Online Supplement, we provide additional robustness checks. We consider increasing the number of groups k in the k-means algorithm from the baseline value of 10 up to 50 in increments of 10, finding that the estimates are nearly identical across k. We also present estimates for two different time periods (2001-2008 and 2008-2015), finding that the worker quality, firm effects, and sorting components change little over time.

E. Online Appendix: Additional Tables and Figures

	Workers		Firms	
Panel A.	**Baseline Sample**			
	Unique	**Observation-Years**	**Unique**	**Observation-Years**
Full Sample:	89,570,480	447,519,609	6,478,231	39,163,975
Panel B.	**Movers Sample**			
	Unique	**Observation-Years**	**Unique**	**Observation-Years**
Movers Only:	32,070,390	207,990,422	3,559,678	23,321,807
Panel C.	**Stayers Sample**			
	Unique	**6 Year Spells**	**Unique**	**6 Year Spells**
Complete Stayer Spells:	10,311,339	35,123,330	1,549,190	6,533,912
10 Stayers per Firm:	6,297,042	20,354,024	144,412	597,912
10 Firms per Market:	5,217,960	16,506,865	117,698	476,878

Table A.1—: Overview of the Sample

Notes: This table provides an overview of the full sample, movers sample, and stayers sample, including the steps involved in defining the stayers sample.

	Goods				Services				All
	Midwest	Northeast	South	West	Midwest	Northeast	South	West	All
Panel A.					**Full Sample**				
Observation Counts:									
Number of FTE Worker-Years	42,908,008	26,699,951	40,312,311	31,585,748	69,044,540	62,386,621	103,227,384	71,355,046	447,519,609
Number of Unique FTE Workers	9,318,707	6,088,530	10,215,128	7,712,759	17,314,497	15,167,028	26,519,284	17,949,625	89,570,480
Number of Unique Firms with FTE Workers	294,879	232,717	439,641	329,566	1,051,548	1,054,944	1,908,178	1,314,168	6,478,231
Number of Unique Markets with FTE Workers	1,508	264	1,774	910	4,092	744	4,909	2,492	16,141
Group Counts:									
Mean Number of FTE Workers per Firm	22.1	17.8	16.1	16.3	10.4	9.7	9.5	9.6	11.4
Mean Number of FTE Workers per Market	2,012.9	6,856.7	1,586.3	2,539.3	1,221.0	5,723.0	1,492.8	2,097.7	1,915.1
Mean Number of Firms per Market with FTE Workers	91.3	384.9	98.3	156.0	117.4	588.2	156.6	217.7	167.6
Outcome Variables in Log $:									
Mean Log Wage for FTE Workers	10.76	10.81	10.70	10.81	10.61	10.74	10.62	10.70	10.69
Mean Value Added for FTE Workers	17.36	16.80	16.68	16.64	16.18	16.04	15.94	16.07	16.31
Firm Aggregates in $1,000:									
Wage Bill per Worker	43.6	50.7	42.2	52.9	34.1	44.2	35.8	40.3	40.8
Value Added per Worker	91.2	107.5	85.2	91.7	90.5	111.1	94.2	92.3	95.2
Panel B.					**Movers Sample**				
Observation Counts:									
Number of FTE Mover-Years	17,455,849	11,543,303	18,066,928	15,513,020	31,643,497	28,390,782	50,052,742	35,324,301	207,990,422
Number of Unique FTE Movers	4,124,895	2,829,881	4,819,645	3,876,182	7,723,804	6,662,132	11,904,098	8,321,469	32,070,390
Number of Unique Firms with FTE Movers	188,376	144,268	265,374	215,092	571,360	549,064	1,018,957	700,618	3,559,678
Number of Unique Markets with FTE Movers	1,457	261	1,747	872	3,899	739	4,766	2,342	15,586
Group Counts:									
Mean Number of FTE Movers per Firm with FTE Movers	13.5	11.9	11.2	11.6	8.2	7.9	7.9	8.2	8.9
Mean Number of Movers per Market with FTE Movers	864.8	2,991.3	732.4	1,318.1	599.3	2,655.3	761.5	1,123.7	940.6
Mean Number of Firms per Market with FTE Movers	64.1	251.1	65.5	113.4	72.7	337.1	96.4	137.7	105.5
Outcome Variables in Log $:									
Mean Log Wage for FTE Movers	10.68	10.77	10.64	10.78	10.59	10.72	10.61	10.70	10.67
Mean Value Added for FTE Movers	16.72	16.52	16.28	16.36	16.04	16.02	15.88	16.01	16.12
Panel C.					**Stayers Sample**				
Sample Counts:									
Number of 8-year Worker-Firm Stayer Spells	2,588,628	1,777,928	1,237,821	1,150,115	2,315,238	2,527,212	2,609,997	2,207,552	16,506,865
Number of Unique FTE Stayers in Firms with 10 FTE Stayers	798,575	532,507	416,549	354,518	740,091	764,699	865,629	724,155	5,217,960
Number of Unique Firms with 10 FTE Stayers	13,884	10,896	9,409	9,767	18,083	19,475	19,626	16,185	117,698
Number of Unique Markets with 10 Firms with 10 FTE Stayers	197	111	216	104	335	213	438	219	1,826
Outcome Variables in Log $:									
Mean Log Wage for FTE Stayers	10.95	10.99	10.97	10.99	10.90	11.01	10.96	11.05	10.97
Mean Log Value Added for FTE Stayers	18.04	17.56	17.46	16.56	17.45	17.23	17.89	17.93	17.61

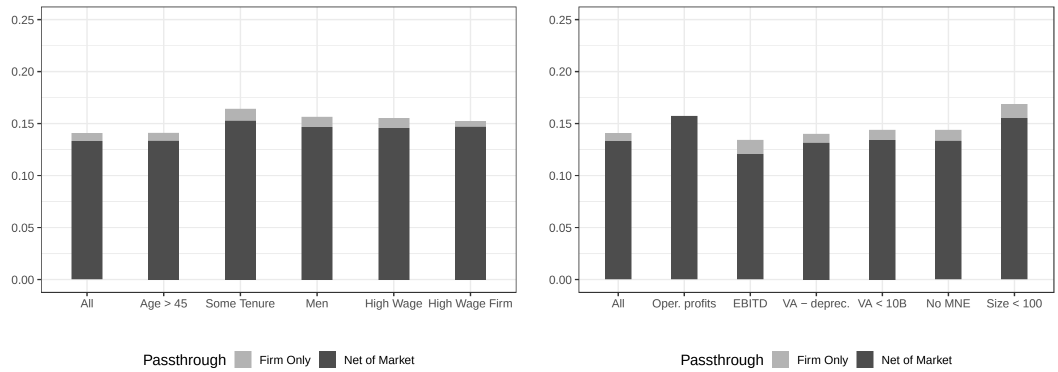
Table A.2—: Detailed sample characteristics

Notes: This table provides detailed sample characteristics for the full sample, movers sample, and stayers sample.

	GMM Estimates of Joint Process			
	Firm Only		Accounting for Markets	
	Log Value Added	Log Earnings	Log Value Added	Log Earnings
Panel A.	**Process: MA(1)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
Transitory Shock (Std. Dev.)	0.18	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.09	0.15	0.09	0.15
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 2	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Permanent Passthrough Coefficient		0.14		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.01		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.02)
Panel B.	**Process: MA(2)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.00)	(0.00)
Transitory Shock (Std. Dev.)	0.17	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.05	0.21	0.07	0.21
	(0.05)	(0.01)	(0.04)	(0.01)
MA Coefficient, Lag 2	-0.03	0.04	-0.01	0.04
	(0.03)	(0.00)	(0.02)	(0.00)
Permanent Passthrough Coefficient		0.15		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.02		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.03)

Table A.3—: GMM estimates of the earnings and value added processes

Notes: This table displays the parameters of the joint processes of log value added and log earnings. These results come from joint estimation of the earnings and value added processes under assumptions 1-3 using GMM. Columns 1-2 report results from the specification which imposes $\gamma_r = \Upsilon$ ("Firm only"), while columns 3-4 report results from the specification which allows Υ to differ from γ_r ("Accounting for Markets"). The top panel assumes the transitory components follow an MA(1) process. The bottom panel permits the transitory components to follow an MA(2) process. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.



(a) Heterogeneity across Worker Samples

(b) Heterogeneity across Firm Samples and Value Added Measures

Figure A.1. : Sample heterogeneity in pass-through rates of firm shocks

Notes: This figure displays heterogeneity in the GMM estimates of the pass-through rates of a firm shock, both for the firm only model (imposing $\Upsilon = \gamma$) and when removing market by year means (permitting $\Upsilon \neq \gamma$).

Outcome Sample	First Stage (Std. Error)	Reduced Form (Std. Error)	Second Stage (Std. Error)
Procurement auction shock at firm-level			
8,677 unique auction bidders	0.143	0.020	0.142
	(0.039)	(0.006)	(0.068)
Shift-share industry value added shock			
667 unique commuting zones	0.708	0.134	0.189
	(0.216)	(0.061)	(0.041)

Table A.4—: Additional details regarding pass-through estimation using external instruments

Notes: This table provides additional details on the pass-through estimation using external instruments.

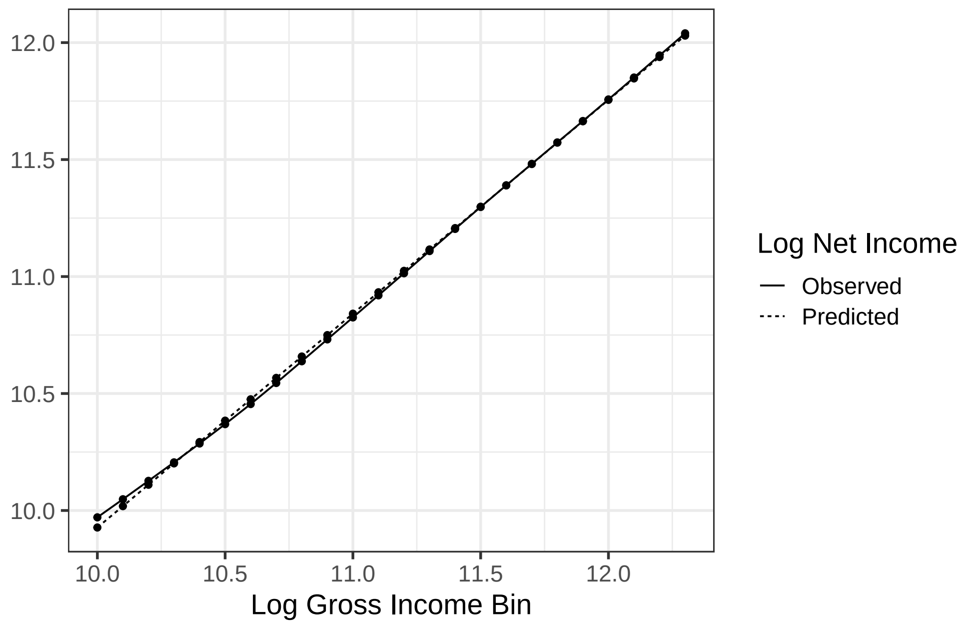


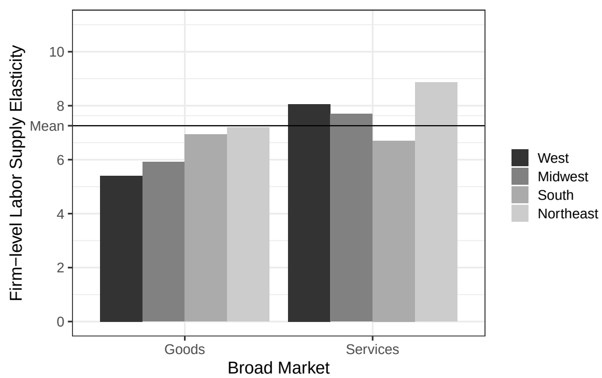
Figure A.2. : Fit of the Tax Function

Notes: In this figure, we display the log net income predicted by the tax function compared to the log net income observed in the data.

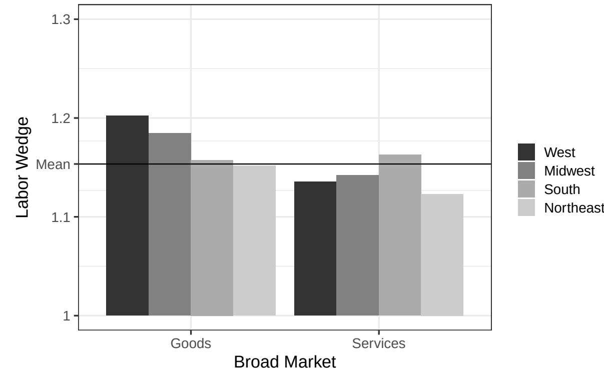
	Market Count (in 1,000)		Passthrough Rate		Average of the Model Parameters			Workers' Share of Rents Firm-level	Workers' Share of Rents Market-level
	Workers	Firms	Market	Firm	β	$1-\rho_r^2$	$1-\alpha_r$	$\frac{R^w}{R^w+R^f}$	$\frac{R^{wm}}{R^{wm}+R^{fm}}$
Baseline (NAICS 2-digit, commuting zone)	1.90	0.17	0.18	0.13	4.99	0.51	0.79	0.52	0.50
Shutdown broad market heterogeneity ($\rho_r = \overline{\rho}, \alpha_r = \overline{\alpha}$)	1.97	0.17	0.18	0.13	5.06	0.48	0.79	0.52	0.51
Alternative detailed markets:									
Finer geography (county)	0.54	0.05	0.19	0.14	4.61	0.54	0.79	0.51	0.49
Finer industry (NAICS 3-digit)	0.65	0.06	0.19	0.13	4.60	0.59	0.79	0.52	0.50
Coarser geography (state)	25.44	2.23	0.18	0.13	5.00	0.52	0.79	0.53	0.50
Coarser industry (NAICS supersector)	4.42	0.39	0.20	0.13	4.28	0.66	0.79	0.53	0.51

Table A.5—: Robustness of the Model Parameters and Rent Sharing Estimates to Alternative Market Definitions

Notes: This table displays robustness of the estimated model parameters and rents to alternative definitions of detailed markets.



(a) Labor Supply Elasticity $\frac{\beta}{\rho_r \lambda}$



(b) Labor Wedge $1 + \frac{\rho_r \lambda}{\beta}$

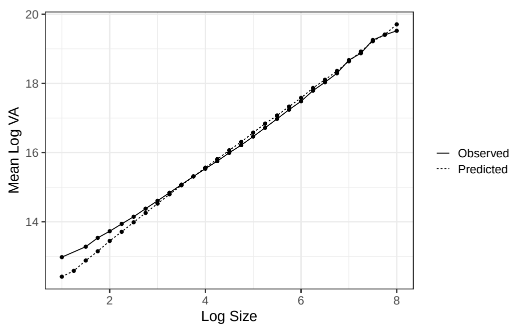
Figure A.3. : Broad Market Heterogeneity in Labor Supply Elasticities and Labor Wedges

Notes: In this figure, we display the estimated (post-tax) firm level labor supply elasticity and labor wedge for each of the 8 broad markets. The overall worker-weighted means are represented by horizontal lines.

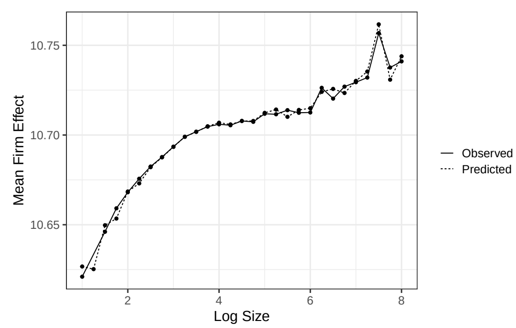
		Model Specifications			
		Main	**Alternatives**		
			$\theta_j = \bar{\theta}$	$\gamma_r = \Upsilon = 0$	$\theta_j = \bar{\theta}$ and $\gamma_r = \Upsilon = 0$
Share explained by:					
i) Worker Quality	$Var(\tilde{x}_i)$	71.6%	73.5%	70.4%	72.4%
ii) Firm Effects	$Var(\tilde{\psi}_{j(i)})$	4.3%	3.0%	4.3%	3.2%
iii) Sorting	$2Cov(\tilde{x}_i, \tilde{\psi}_{j(i)})$	13.0%	12.8%	13.1%	12.9%
iv) Interactions	$Var(\varrho_{ij}) + 2Cov(x_i + \psi_{j(i)}, \varrho_{ij})$	0.9%		1.2%	
v) Time-varying Effects	$Var(\psi^a_{j(i),t}) + 2Cov(x_i, \psi^a_{j(i),t})$	0.3%	0.3%		
Sorting Correlation:	$Cor(x_i, \psi_{j(i)})$	0.37	0.43	0.38	0.43
Variance Explained:	R^2	0.90	0.90	0.89	0.89
Specification:					
Firm-Worker Interactions		✓	✗	✓	✗
Time-varying Firm Effects		✓	✓	✗	✗

Table A.6—: Decomposition of earnings inequality

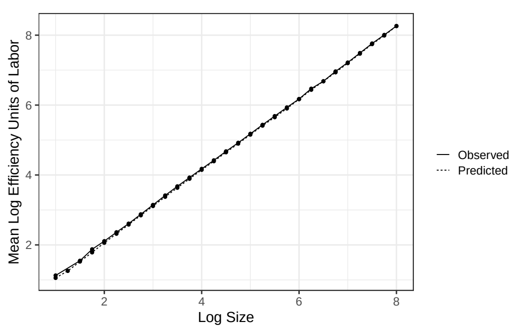
Notes: This table presents the decomposition of log earnings variation into firm and worker effects using the main specification described in the text, as well as alternative specifications that ignore firm-worker interactions ($\theta_j = \bar{\theta}$), ignore time-varying effects ($\gamma_r = \Upsilon = 0$), and ignore both ($\theta_j = \bar{\theta}$ and $\gamma_r = \Upsilon = 0$). The analysis uses both workers who move between firms and non-movers. All estimates are corrected for limited mobility bias using the grouped fixed-effect method of Bonhomme, Lamadon and Manresa (2019).



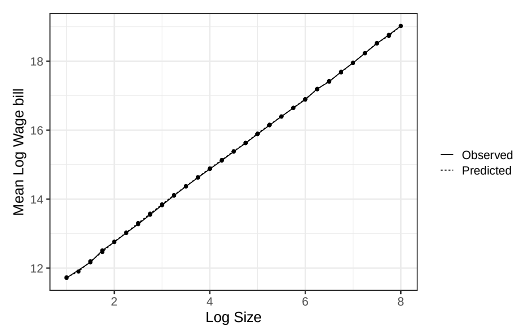
(a) Value Added



(b) Firm Effects



(c) Efficiency Units of Labor



(d) Wage Bill

Figure A.4. : Fit of the Model for Untargeted Moments

Notes: In this figure, we compare the observed and the predicted values of firm effects, value added, efficiency units of labor, and wage bill. We make this comparison separately according to actual and predicted firm size.

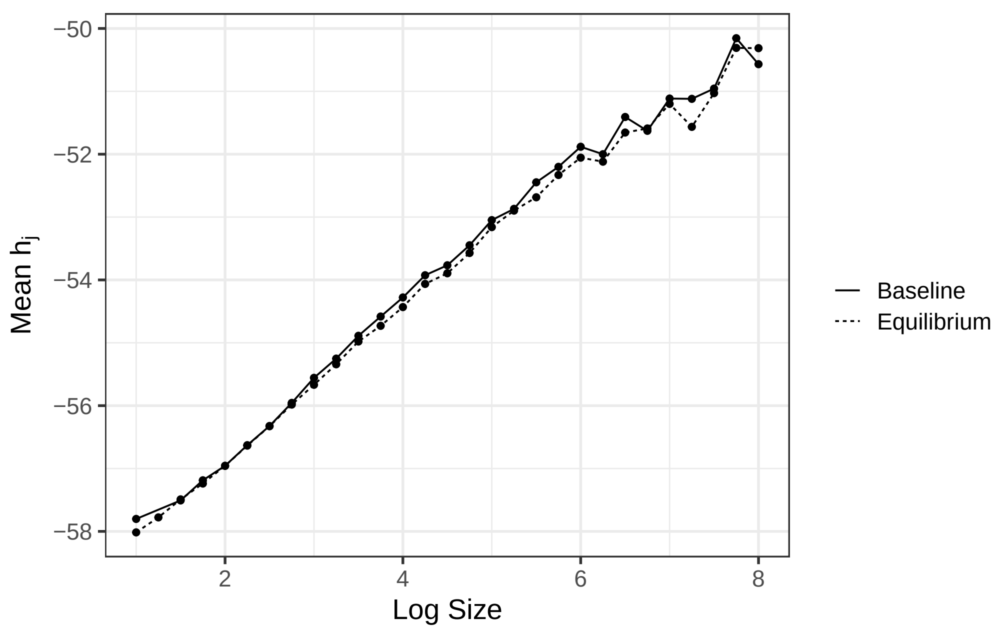


Figure A.5. : Estimates of the Amenity Components h_j from the Wage Equation versus the Equilibrium Constraint

Notes: In this figure, we plot the mean of h_j across log size bins. We compare the baseline estimates of h_j from the equation for firm wage premiums (15), versus those estimated using the equilibrium constraint by solving the fixed-point definition of h_j as a function of $(\tilde{P}_j, \bar{P}_r, G_j(X))$, as shown in Lemma 3.

	Goods				Services			
	Midwest	Northeast	South	West	Midwest	Northeast	South	West
Panel A.	**Model Parameters**							
Idyosinctratic taste parameter (β^{-1})	0.200							
	(0.044)							
Taste correlation parameter (ρ)	0.844	0.694	0.719	0.924	0.649	0.563	0.744	0.619
	(0.179)	(0.153)	(0.160)	(0.182)	(0.141)	(0.109)	(0.246)	(0.117)
Returns to scale ($1-\alpha$)	0.746	0.764	0.863	0.949	0.753	0.740	0.814	0.752
	(0.016)	(0.013)	(0.017)	(0.019)	(0.013)	(0.015)	(0.036)	(0.015)
Panel B.	**Firm-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	6,802	6,681	5,737	8,906	4,234	4,847	5,009	4,805
	(770)	(723)	(720)	(867)	(502)	(803)	(1,295)	(684)
Share of Earnings	16%	13%	14%	17%	12%	11%	14%	12%
	(2%)	(1%)	(2%)	(2%)	(1%)	(2%)	(4%)	(2%)
Firms' Rents:								
Per-worker Dollars	4,041	4,198	7,465	20,069	3,531	3,097	6,915	3,018
	(1,243)	(1,130)	(2,681)	(6,323)	(1,004)	(1,305)	(5,650)	(1,060)
Share of Profits	8%	7%	17%	52%	6%	5%	12%	6%
	(3%)	(2%)	(6%)	(16%)	(2%)	(2%)	(10%)	(2%)
Workers' Share of Rents	63%	61%	43%	31%	55%	61%	42%	61%
	(4%)	(4%)	(5%)	(4%)	(4%)	(5%)	(9%)	(5%)
Panel C.	**Market-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	7,837	9,102	7,572	9,506	6,115	7,935	6,422	7,230
	(1,319)	(1,532)	(1,274)	(1,600)	(1,029)	(1,335)	(1,081)	(1,217)
Share of Earnings	18%	18%	18%	18%	18%	18%	18%	18%
	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)
Firms' Rents:								
Per-worker Dollars	4,940	6,311	10,000	20,846	5,734	5,897	9,363	5,153
	(1,140)	(1,350)	(2,267)	(5,787)	(1,351)	(1,786)	(4,218)	(1,433)
Share of Profits	10%	11%	23%	54%	10%	9%	16%	10%
	(2%)	(2%)	(5%)	(15%)	(2%)	(3%)	(7%)	(3%)
Workers' Share of Rents	61%	59%	43%	31%	52%	57%	41%	58%
	(3%)	(3%)	(4%)	(5%)	(3%)	(4%)	(8%)	(4%)

Table A.7—: Market Heterogeneity in Model Parameters and Rent Sharing Estimates

Notes: This table displays heterogeneity in the estimated model parameters and rents. These results correspond to the specification which allows Υ to differ from γ, and for ρ_r and α_r to vary across broad markets. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.

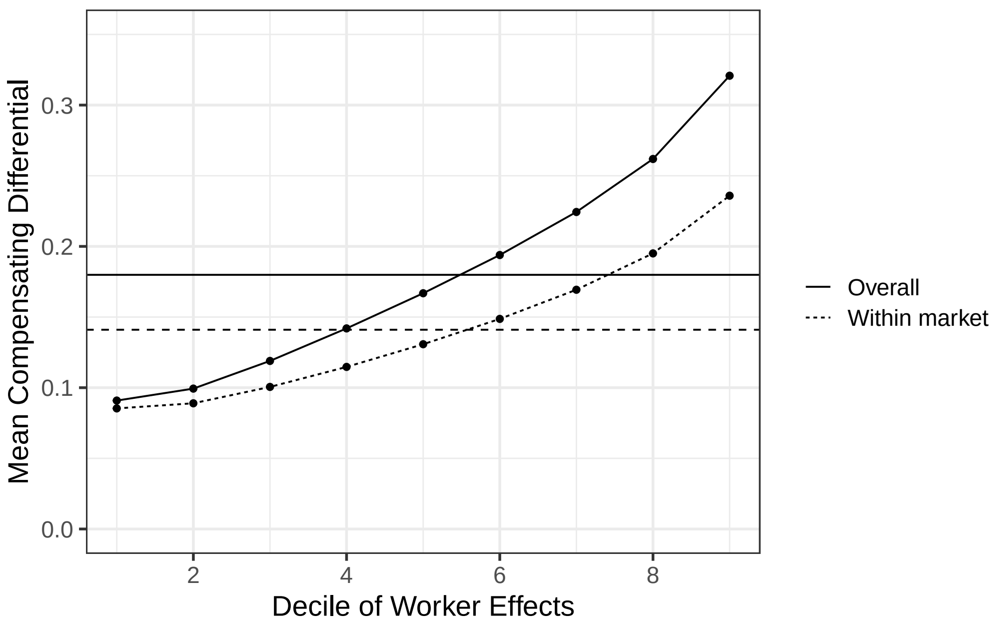
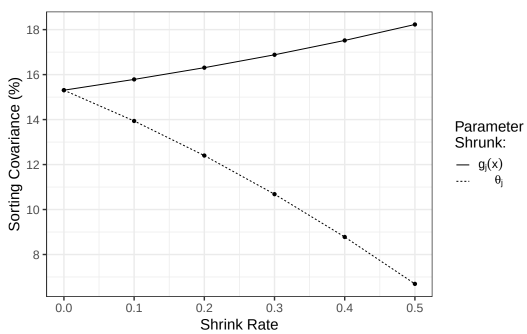
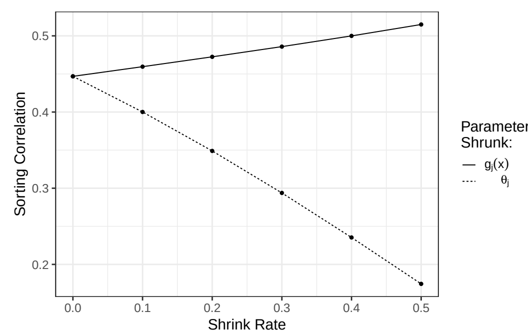


Figure A.6. : Compensating differentials

Notes: In this figure, we plot mean compensating differentials overall and within market. To do so, we randomly draw a pair of firms (j, j') with probability proportional to size. Each j' is drawn from the full set of firms when estimating overall compensating differentials and from the set of firms in the same market as j when estimating within-market compensating differentials. Then, we estimate the compensating differential between j and j' for a worker of given quality $x_i = x$ by $\psi_{j'} + x\theta_{j'} - \psi_j - x\theta_j$. This figure plots the mean absolute value of the compensating differentials across deciles of the x_i distribution, where the horizontal lines denote means across the distribution of x_i.



(a) Sorting



(b) Sorting Correlation

Figure A.7. : Worker sorting with counterfactual values of $g_j(x)$ and θ_j

Notes: In this figure, we reduce the heterogeneity across firms in amenities or production complementarities by replacing either $g_j(x)$ with $(1-s)\,g_j(x) + s\overline{g}_j$ or θ_j with $(1-s)\,\theta_j + s\bar{\theta}$, where $\overline{g}_j = \mathbb{E}_x\,[g_j(x)]$, $\bar{\theta} = \mathbb{E}\,[\theta_j]$. Here, $s \in [0,1]$ is the shrink rate with $s = 0$ corresponding to the baseline model. We report the share of log earnings variance explained by sorting (subfigure a) and the sorting correlation (subfigure b).

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